Unit Overview
In this unit, you will build linear models and use them to study functions, domain, and range. Linear models are the foundation for studying slope as a rate of change, intercepts, and direct variation. You will learn to write linear equations given varied information and express these equations in different forms.

Key Terms
As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary
- causation

Math Terms
- relation
- function
- vertical line test
- independent variable
- dependent variable
- continuous
- discrete
- y-intercept
- relative maximum
- relative minimum
- extrema
- x-intercept
- parent function
- absolute value function
- direct variation
- constant of variation
- indirect variation
- inverse function
- one-to-one
- arithmetic sequence
- explicit formula
- recursive formula
- slope-intercept form
- point-slope form
- standard form
- scatter plot
- trend line
- correlation
- line of best fit
- linear regression
- quadratic regression
- quadratic function
- exponential regression
- exponential function

Embedded Assessments
This unit has three embedded assessments, following Activities 8, 11, and 13. They will give you an opportunity to demonstrate what you have learned.

Embedded Assessment 1:
Representations of Functions p. 121

Embedded Assessment 2:
Linear Functions and Equations p. 173

Embedded Assessment 3:
Linear Models and Slope as Rate of Change p. 207
Write your answers on notebook paper. Show your work.

1. Copy and complete the table of values.

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<thead>
<tr>
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<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>2</td>
<td>5</td>
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<tr>
<td>5</td>
<td>11</td>
<td>8</td>
<td>23</td>
</tr>
<tr>
<td>11</td>
<td>29</td>
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</table>

2. List the integers that make this statement true.

\(-3 \leq x < 4\)

3. Evaluate for \(a = 3\) and \(b = -2\).

a. \(2a - 5\)

b. \(3b + 4a\)

4. Name the point for each ordered pair.

a. \((-3, 0)\)

b. \((-1, 3)\)

c. \((2, -2)\)

5. Explain how you would plot \((3, -4)\) on a coordinate plane.

6. Which of the following equations represents the data in the table?

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</tbody>
</table>

A. \(y = 2x - 1\)

B. \(y = 3x - 1\)

C. \(y = x + 1\)

D. \(y = 2x + 1\)

7. If \(2x + 6 = 2\), what is the value of \(x\)?

A. 4

B. 2

C. 0

D. -2

8. Which of the following are the coordinates of a point on this line?

A. \((-1, 3)\)

B. \((1, -3)\)

C. \((-1, -3)\)

D. \((1, 3)\)
Learning Targets:
- Represent relations and functions using tables, diagrams, and graphs.
- Identify relations that are functions.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Think-Pair-Share, Interactive Word Wall, Paraphrasing

Use this machine to answer the questions below.

1. What DVD would you receive if you inserted your money and pressed:
   a. A1?
   b. C2?
   c. B3?

2. Assuming the machine were filled properly, describe what would happen if you pressed the same button twice.
Each time you press a button, an **input**, you may receive a DVD, an **output**.

3. In the DVD vending machine situation, does every input have an output? Explain your response.

4. Each combination of input and output can be expressed as a **mapping** written \( \text{input} \to \text{output} \). For example, B2 \( \to \) The Amazing Insectman.

a. Write as mappings each of the possible combinations of buttons pushed and DVDs received in the vending machine.

b. Create a table to illustrate how the inputs and outputs of the vending machine are related.

Mappings that relate values from one set of numbers to another set of numbers can be written as **ordered pairs**. A **relation** is a set of ordered pairs. Relations can have a variety of representations. Consider the relation \( \{(1, 4), \)
Lesson 5-1
Relations and Functions

Relations can have a variety of representations. Consider the relation \{(1, 4), (2, 3), (6, 5)\}, shown here as a set of ordered pairs. This relation can also be represented in these ways.

<table>
<thead>
<tr>
<th>Table</th>
<th>Mapping</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(y)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

5. Write the following numerical mappings as ordered pairs.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Ordered Pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\rightarrow) 2</td>
<td>(1, -2)</td>
</tr>
<tr>
<td>2</td>
<td>(\rightarrow) 1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(\rightarrow) 4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(\rightarrow) 7</td>
<td></td>
</tr>
</tbody>
</table>

Check Your Understanding

6. A vending machine at the Ocean, Road, and Air show creates souvenir coins. You select a letter and a number and the machine creates a souvenir coin with a particular vehicle imprinted on it. The graph shows the vending machine letter/number combinations for the different coins.

a. Make a table showing each coin’s letter/number combination.
b. Write the letter/number combinations as a set of ordered pairs.
c. Write the letter/number combinations in a mapping diagram.
A *function* is a relation in which each input is paired with exactly one output.

7. Compare and contrast the DVD Vending Machine with a function.

8. Suppose when pressing button C1 on the vending machine both “The Dependables” and “The Light Knight” come out. Describe how this vending machine resembles or does not resemble a function.

9. Imagine a machine where you input an age and the machine gives you the name of anyone who is that age. Compare and contrast this machine with a function. Explain by using examples and create a representation of the situation.

10. Create an example of a situation (math or real-life) that behaves like a function and another that does not behave like a function. Explain why you chose each example to fit the category.
   a. Behaves like a function:

   b. Does not behave like a function:
Lesson 5-1
Relations and Functions

11. Determine whether the ordered pairs and equations represent functions. Explain your answers.
   a. \{(5, 4), (6, 3), (7, 2)\}
   b. \{(4, 5), (4, 3), (5, 2)\}
   c. \{(5, 4), (6, 4), (7, 4)\}
   d. \(y = 3x - 5\), where \(x\) represents input values and \(y\) represents output values
   e. \(y = -x + 4\), where \(x\) represents input values and \(y\) represents output values

12. Attend to precision. Using positive integers, write two relations as lists of ordered pairs below, one that is a function and one that is not a function.

   Function:
   Not a function:

Check Your Understanding

13. Does the mapping shown represent a function? Explain.

14. Does the graph shown represent a function? Explain.
LESSON 5-1 PRACTICE

For the Bingo card below, suppose that a combination of a column letter and a row number, such as B1, represents an input and the number at that location, such as 7, represents an output. Use this information for Items 15–17.

15. What output corresponds to I2?
16. What input corresponds to 54?
17. Does every input have a numerical output? Explain.
18. Construct viable arguments. Explain why each of the following is not a function.
   a. 
   b. 
   c. $y^2 = x$, where $x$ represents input values and $y$ represents output values.
Learning Targets:
- Describe the domain and range of a function.
- Find input-output pairs for a function.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Create Representations, Discussion Groups, Marking the Text, Sharing and Responding

The set of all inputs for a function is known as the **domain** of the function. The set of all outputs for a function is known as the **range** of the function.

1. Consider a vending machine where inserting 25 cents dispenses one pencil, inserting 50 cents dispenses 2 pencils, and so forth up to and including all 10 pencils in the vending machine.
   a. Identify the domain in this situation.
   b. Identify the range in this situation.

2. For each function below, identify the domain and range.
   a. | input | output |
      | 7     | 6      |
      | 3     | −2     |
      | 5     | 1      |
   
   Domain: 
   Range: 
   
   b. ![Graph](image)
      
      Domain: 
      Range: 
   
   c. ![Graph](image)
      
      Domain: 
      Range: 
   
   d. \{ (−7, 0), (9, −3), (−6, 2.5) \}
      
      Domain: 
      Range: 

The **domain** and **range** of a function can be written using set notation.

For example, for the function \{(1, 2), (3, 4), (5, 6)\}, the domain is \{1, 3, 5\} and the range is \{2, 4, 6\}.
3. Consider a machine that exchanges quarters for dollar bills. Inserting one dollar bill returns four quarters and you may insert up to five one-dollar bills at a time.
   a. Is 7 a possible input for the relation this change machine represents? Justify your response.

   b. Could 3.5 be included in the domain of this relation? Explain why or why not.

   c. Reason abstractly. What values are not in the domain? Justify your reasoning.

   d. Is 8 a possible output for the relation this change machine represents? Justify your response.

   e. Could 3 be included in the range of this relation? Explain why or why not.

   f. What values are not in the range? Justify your reasoning.
4. **Make sense of problems.** Each of the functions that you have seen has a *finite* number of ordered pairs. There are functions that have an *infinite* number of ordered pairs. Describe any difficulties that may exist trying to represent a function with an infinite number of ordered pairs using the four representations of functions that have been described thus far.

5. Sometimes, machine diagrams are used to represent functions. In the function machine below, the inputs are labeled \( x \) and the outputs are labeled \( y \). The function is represented by the expression \( 2x + 5 \).

\[
\begin{array}{c}
\text{x} \\
\downarrow \\
2x + 5 \\
\downarrow \\
\text{y}
\end{array}
\]

a. What is the output if the input is \( x = 7 \)? \( x = -2 \)? \( x = \frac{1}{2} \)?

b. **Express regularity in repeated reasoning.** Is there any limit to the number of input values that can be used with this expression? Explain.

Consider the function machine below.

\[
\begin{array}{c}
\text{x} \\
\downarrow \\
x^2 + 2x + 3 \\
\downarrow \\
\text{y}
\end{array}
\]

6. Use the diagram to find the (input, output) ordered pairs for the following values.

a. \( x = -5 \)  
   b. \( x = \frac{3}{5} \)  
   c. \( x = -10 \)
Lesson 5-2
Domain and Range

7. Make a function machine for the expression $10 - 5x$. Use it to find ordered pairs for $x = 3, x = -6, x = 0.25,$ and $x = \frac{3}{4}$.

Creating a function machine can be time consuming and awkward. The function represented by the diagram in Item 5 can also be written algebraically as the equation $y = 2x + 5$.

8. For each function, find ordered pairs for $x = -2, x = 5, x = \frac{2}{3},$ and $x = 0.75$. Create tables of values.

a. $y = 9 - 4x$

b. $y = \frac{1}{x}$

Check Your Understanding

9. The set $\{(3, 5), (-1, 2), (2, 2), (0, -1)\}$ represents a function. Identify the domain and range of the function.

10. Identify the domain and range for each function.

a. [Diagram of a coordinate plane with points marked]

b. [Table]

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>-8</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>-4</td>
<td>9</td>
</tr>
</tbody>
</table>
Lesson 5-2
Domain and Range

LESSON 5-2 PRACTICE

Identify the domain and range.

11. 

12. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
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<tbody>
<tr>
<td>1.5</td>
<td>4</td>
</tr>
<tr>
<td>−0.3</td>
<td>8</td>
</tr>
<tr>
<td>1/6</td>
<td>3</td>
</tr>
</tbody>
</table>

13. **Model with mathematics.** At an arcade, there is a machine that accepts game tokens and returns tickets that can be redeemed for prizes. Inserting 5 tokens returns 3 tickets and inserting 10 tokens returns 8 tickets. You must insert tokens in multiples of 5 or 10, and you have a total of 20 tokens.
   a. Identify the domain in this situation.
   b. Identify the range in this situation.

14. For the function machine shown, copy and complete the table of values.

15. For each function below, find ordered pairs for \( x = −1 \), \( x = 3 \), \( x = \frac{1}{2} \), and \( x = 0.4 \). Write your results as a set of ordered pairs.
   a. \( y = 4x \)       b. \( y = 2 − x^2 \)
Learning Targets:

- Use and interpret function notation.
- Evaluate a function for specific values of the domain.

SUGGESTED LEARNING STRATEGIES: Create Representations, Discussion Groups

When referring to the functions in Item 8 in Lesson 5-2, it can be confusing to distinguish among them since each begins with “y =.” Function notation can be used to help distinguish among different functions.

For instance, the function \(y = 9 - 4x\) in Item 8a can be written:

\[
\text{This is read as “f of x” and } f(x) \text{ is equivalent to } y.
\]

\[
f(x) = 9 - 4x
\]

1. To distinguish among different functions, it is possible to use different names. Use the name \(h\) to write the function from Item 8b using function notation.

Function notation is useful for evaluating functions for multiple input values. To evaluate \(f(x) = 9 - 4x\) for \(x = 2\), you substitute 2 for the variable \(x\) and write \(f(2) = 9 - 4(2)\). Simplifying the expression yields \(f(2) = 1\).

2. Use function notation to evaluate \(f(x) = 9 - 4x\) for \(x = 5, x = -3,\) and \(x = 0.5\).
Lesson 5-3
Function Notation

3. Use the values for $x$ and $f(x)$ from Item 2. Display the values using each representation.
   a. list of ordered pairs
   b. table of values
   c. mapping
   d. graph

4. Given the function $f(x) = 9 - 4x$ as shown above, what value of $x$ results in $f(x) = 1$?

5. Evaluate each function for $x = -5$ and $x = \frac{4}{3}$.
   a. $f(x) = 2x - 7$
   b. $g(x) = 6x - x^2$
   c. $h(x) = \frac{2}{x^2}$

6. Reason quantitatively. Recall the money-changing machine from Item 3 in Lesson 5-2, in which customers can insert up to five one-dollar bills at a time and receive an equivalent amount of quarters. The function $f(x) = 4x$ represents this situation. What does $x$ represent? What does $f(x)$ represent?
A function whose domain is the set of positive consecutive integers forms a sequence. The terms of the sequence are the range values of the function. For the sequence 4, 7, 10, 13, …, \( f(1) = 4 \), \( f(2) = 7 \), \( f(3) = 10 \), and \( f(4) = 13 \).

7. Consider the sequence \(-4, -2, 0, 2, 4, 6, 8, \ldots\).
   a. What is \( f(3) \)?
   b. What is \( f(7) \)?

8. Evaluate the functions for the domain values indicated.
   a. \( p(x) = 3x + 14 \) for \( x = -5, 0, 4 \)
   b. \( h(t) = t^2 - 5t \) for \( t = -2, 0, 5, 7 \)

9. Consider the sequence \(-7, -3, 1, 5, 9, \ldots\).
   a. What is \( f(2) \)?
   b. What is \( f(5) \)?

**LESSON 5-3 PRACTICE**

Use the function \( y = x^2 - 3x - 4 \) for Items 10–12.

10. Write the function in function notation.

11. Evaluate the function for \( x = -2 \). Express your answer in function notation.

12. **Make use of structure.** For what value of \( x \) does \( f(x) = -4 \)?

13. Consider the sequence \( \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots \). What is \( f(4) \)?
ACTIVITY 5 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 5-1
Use the Beverage Vending Machine to answer Items 1—6.

1. List all of the possible inputs.
2. List all of the possible outputs.
3. Which output results from an input of 2C?
   A. Juice
   B. Iced tea
   C. Latte
   D. Cocoa
4. Which number/letter combination would you input if you wanted the machine to output juice?
   A. 2A
   B. 1B
   C. 2B
   D. 1D
5. In a mapping of the relation shown by the vending machine, what drink would 1D map to?
6. In a table of the relation shown by the vending machine, what number/letter combination would correspond to cocoa?

For Items 7—9, two relations are given. One relation is a function and one is not. Identify each and explain.

7. \{(5, -2), (2, -5), (2, 5), (-5, 2)\}
8. \{(5, -2), (-2, 5), (5, 2), (-5, 2)\}

9. (0, 5) (1, 5) (2, 6) (x, 7)
11. Does the graph shown represent a function? Explain.

Lesson 5-2
Use the graph for Items 12—14.

12. Identify the domain of the relation represented in the graph.
13. Identify the range of the relation represented in the graph.
14. Does the relation shown in the graph represent a function? Explain.

Lesson 5-3
Use the function machine for Items 15—17.

15. How would you write the function shown in the function machine in function notation?
16. What is the value of $f(-2)$?
17. What value(s) of $x$ results in $f(x) = 8$?
18. Given the function $f(x) = -2x - 5$, determine the value of $f(-3)$.

The first seven numbers in the Fibonacci sequence are: 0, 1, 1, 2, 3, 5, 8. Use this information for Items 19 and 20.

19. What is $f(2)$?
20. What is $f(6)$?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

21. Dora said that the mapping diagram below does not represent a function because each value in the domain is paired with the same value in the range. Explain the error in Dora's reasoning.
Roller coasters can be scary but fun to ride. Below is the graph of the heights reached by the cars of the Thunderball Roller Coaster over its first 1250 feet of track. The graph displays a function because each input value has one and only one output value. You can see this visually using the **vertical line test**. Study this graph to determine the domain and range.

The domain gives all values of the *independent variable*: in this case, the distance along the track in feet. The domain values are graphed along the horizontal or x-axis. The domain of the function above can be written in set notation as:

\[ \{ \text{all real values of } x \colon 0 \leq x \leq 1250 \} \]

Read this notation as: *the set of all real values of* \( x \), *between 0 and 1250, inclusive*.

The range gives the values of the *dependent variable*: in this case, the height of the roller coaster above the ground in feet. The range values are graphed on the vertical or y-axis. The range of the function above can be written in set notation as:

\[ \{ \text{all real values of } y \colon 10 \leq y \leq 110 \} \]

Read this notation as: *the set of all real values of* \( y \), *between 10 and 110, inclusive*.
The graph above shows data that are \textbf{continuous}. The points in the graph are connected, indicating that domain and range are sets of real numbers with no breaks in between. A graph of \textbf{discrete} data consists of individual points that are not connected by a line or curve.

Many other useful pieces of information about a function can be determined by looking at its graph.

- The \textbf{y-intercept} of a function is the point at which the graph of the function intersects the \(y\)-axis. The \(y\)-intercept is the point at which \(x = 0\).
- A \textbf{relative maximum} of a function \(f(x)\) is the greatest value of \(f(x)\) for values in a limited open domain interval.
- A \textbf{relative minimum} of a function \(f(x)\) is the least value of \(f(x)\) for values in a limited open domain interval.

Because they must occur within open intervals of the domain, relative maximums and relative minimums cannot correspond to the endpoints of graphs.

Use the Thunderball Roller Coaster Graph on the previous page for Items 1–5.

1. \textbf{Reason abstractly.} What is the \(y\)-intercept of the function shown in the graph, and what does it represent?

2. Identify a relative maximum of the function represented by the graph.

3. Identify the absolute maximum of the function represented by the graph. Interpret its meaning in the context of the situation.

4. Identify a relative minimum of the function represented by the graph.

5. Identify the absolute minimum of the function represented by the graph. Interpret its meaning in the context of the situation.
Suppose you got on a roller coaster called Cougar Mountain that immediately started climbing the track in a linear fashion, as shown in the graph.

6. Identify the domain and range of the function.

7. Identify the $y$-intercept of the function.

8. Identify the absolute maximum and minimum of the function.

9. Does the function have any relative maximum or minimum values? Explain.

10. How are the extrema different on this linear graph versus the nonlinear graph for the Thunderball Roller Coaster?

**Extrema** refers to all maximum and minimum values.
11. The graph below shows five points that make up the function \( h \). Is the function \( h \) continuous? Explain.

![Graph of function h]

12. A function has three relative maximums: -2, 10.3, and 28. One of the relative maximums is also the absolute maximum. What is the absolute maximum?

Tell whether each statement is sometimes, always, or never true. Explain your answers.

13. A relative minimum is also an absolute minimum.

14. An absolute minimum is also a relative minimum.

Tom hiked along a circular trail known as the Juniper Loop. The graph shows his distance \( d \) from the starting point after \( t \) minutes.

![Graph of distance from start]

15. Identify the domain and range of the function shown in the graph.

16. Identify the absolute minimum of the function. What does it represent?
Lesson 6-1
Key Features of Graphs

17. In this function, the absolute minimum corresponds to two points on the graph. What are the two points? What do they represent in this context?

18. Identify the absolute maximum of the function. What does it represent?

19. What points on the graph correspond to the absolute maximum? What does this mean in the context of Tom's hike?

20. Identify any relative minimums for the function shown in the graph.

21. Identify any relative maximums for the function shown in the graph.

Check Your Understanding

22. What are the independent and dependent variables for the function representing Tom's hike?

23. Explain how to determine the maximum and minimum values of a function by examining its graph.

24. Is it possible for a function to have more than one absolute maximum or absolute minimum value? Explain.
LESSON 6-1 PRACTICE

Model with mathematics. Use the graph below for Items 25–30.

25. What are the independent and dependent variables? Explain.
26. Use set notation to write the domain and range of the function.
27. Is the function discrete or continuous? Explain.
28. What is the $y$-intercept? Interpret the meaning of the $y$-intercept in this context.
29. Identify any relative maximums or minimums of the function.
30. Identify the absolute maximum and absolute minimum values. Interpret their meanings in this context.
Lesson 6-2
More Complex Graphs

Learning Targets:
- Relate the domain and range of a function to its graph and to its function rule.
- Identify and interpret key features of graphs.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Levels of Questions, Think Aloud, Create Representations, Summarizing

Examine the graph of the function \( f(x) = \frac{1}{(x-2)^2} \), graphed below.

1. Describe how this graph is different from the graphs in Lesson 6-1.

Example A
Give the domain and range of the function \( f(x) = \frac{1}{(x-2)^2} \).

Then find the \( y \)-intercept, the absolute maximum, and the absolute minimum.

To find the domain and range:

**Step 1:** Study the graph.
The sketch of this graph is a portion of the function represented by the equation \( f(x) = \frac{1}{(x-2)^2} \).

**Step 2:** Look for values for which the domain causes the function to be undefined. Look how the graph behaves near \( x = 2 \).

**Solution:** The domain and range of \( f(x) = \frac{1}{(x-2)^2} \) can be written:

- Domain: \( \{ \text{all real values of } x : x \neq 2 \} \)
- Range: \( \{ \text{all real values of } y : y > 0 \} \)

**MATH TIP**
Notice the result when \( x = 2 \) is substituted into \( f(x) \).

\[
f(2) = \frac{1}{(2-2)^2} = \frac{1}{0}
\]
Division by zero is undefined in mathematics.
To determine the \(y\)-intercept and identify any maximums or minimums:

Study the graph. We can see that the function intersects the \(y\)-axis at \((0, 0.25)\). The value of \(f(x)\) keeps getting larger as \(x\) approaches 2 from both sides. The value of \(f(x)\) approaches, but never reaches, 0 as \(x\) gets further from 2 on both sides.

**Solution:** The \(y\)-intercept is \((0, 0.25)\). The function does not have an absolute maximum or minimum.

**Try These A**

The function \(f(x) = 8 + 2x - x^2\) is graphed below.

![Graph of \(f(x) = 8 + 2x - x^2\)](image)

**a.** Identify the domain and range of the function.
   Domain: ___________________________
   Range: ___________________________

**b.** Identify the \(y\)-intercept.

**c.** Identify any relative or absolute minimums of the function.

**d.** Identify any relative or absolute maximums of the function.
2. The equation \( y = 2x - 1 \) is graphed below.

![Graph of \( y = 2x - 1 \)]

a. Identify the domain and range.
   Domain: 
   Range: 

b. What is the \( y \)-intercept of \( y = 2x - 1 \)?

c. Identify any relative or absolute minimums of \( y = 2x - 1 \).

d. Identify any relative or absolute maximums of \( y = 2x - 1 \).

e. Construct viable arguments. Explain whether this equation represents a function and how you determined this.

3. The function \( y = 2^x \) is graphed below.

![Graph of \( y = 2^x \)]

a. Identify the domain and range.
   Domain: 
   Range: 

b. What is the \( y \)-intercept of the function \( y = 2^x \)?
c. Identify any relative or absolute minimums of \( y = 2^x \).

d. Identify any relative or absolute maximums of \( y = 2^x \).

4. If you have access to a graphing calculator, work with a partner to graph the equations listed in the table below. Each equation is a function.
   a. Using the graphs you create, determine the domain and range for each function from the possibilities listed below the chart.
   b. Select the appropriate domain from choices 1–6 and record your answer in the Domain column. Then select the appropriate range from choices a–f and record the appropriate range in the Range column.
   c. When the chart is complete, compare your answers with those from another group.

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = -3x + 4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = x^2 - 6x + 5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = 9x - x^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y =</td>
<td>x + 1</td>
<td>)</td>
</tr>
<tr>
<td>( y = 3 + \sqrt{x} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = \frac{4}{x} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Possible Domains**
1) all real numbers
2) all real \( x \), such that \( x \neq -2 \)
3) all real \( x \), such that \( x \neq 0 \)
4) all real \( x \), such that \( x \neq 2 \)
5) all real \( x \), such that \( x \geq 0 \)
6) all real \( x \), such that \( x \leq 0 \)

**Possible Ranges**
1) all real numbers
2) all real \( y \), such that \( y \neq 0 \)
3) all real \( y \), such that \( y \geq -4 \)
4) all real \( y \), such that \( y \geq 0 \)
5) all real \( y \), such that \( y \leq 20.25 \)
6) all real \( y \), such that \( y \geq 3 \)

**MATH TIP**
The domain is restricted to avoid situations where division by zero or taking the square root of a negative number would occur.
Lesson 6-2
More Complex Graphs

Check Your Understanding

5. How can you determine from a function's graph whether the function has any maximum or minimum values?
6. How can you determine the domain of a function by examining its graph? By examining its function rule?
7. Give an example of a function that has a restricted domain. Justify your answer.

LESSON 6-2 PRACTICE

The function \( f(x) = 2x^2 - 3 \) is graphed below.

8. Give the domain, range, and \( y \)-intercept.
9. Identify any relative or absolute minimums.
10. Identify any relative or absolute maximums.
11. Attend to precision. Examine the graphs below. Explain why one function has an absolute minimum and an absolute maximum and the other function does not. Identify the absolute minimum and maximum values of the function for which they exist.
Learning Targets:
- Identify and interpret key features of graphs.
- Determine the reasonable domain and range for a real-world situation.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion, Groups, Look for a Pattern

The function \( f(x) = 3 + 2x \) is graphed below.

1. What are the domain and range of the function?
   Domain:
   Range:

In many real-world situations, not all values make sense for the domain and/or range. For example, distance cannot be negative; number of people cannot be a decimal or a fraction. In such situations, the values that make sense for the domain and range are called the reasonable domain and range.

Example A
A taxi ride costs an initial rate of $3.00, which is charged as soon as you get in the cab, plus $2 for each mile traveled. The cost of traveling \( x \) miles is given by the function \( f(x) = 3 + 2x \). What are the reasonable domain and range?

Step 1: Sketch a graph of the function.

### MATH TIP

Graph a function by substituting several values for \( x \) and generating ordered pairs. You can organize the ordered pairs in a table. There are infinitely many other solutions because the graph has infinitely many points.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = 3 + 2x )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>(1, 5)</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>(2, 7)</td>
</tr>
</tbody>
</table>
Step 2: Determine the reasonable domain. Think about what the variable $x$ represents. What values make sense?

The variable $x$ represents the number of miles, so it does not make sense for $x$ to be negative.

The reasonable domain is \{ $x$: $x \geq 0$ \}.

Step 3: Use the reasonable domain and the graph to determine the reasonable range.

From the graph, all $y$-values corresponding to the reasonable domain values are greater than or equal to 3. The reasonable range is \{ $y$: $y \geq 3$ \}.

**Solution:** The reasonable domain is \{ $x$: $x \geq 0$ \}. The reasonable range is \{ $y$: $y \geq 3$ \}.

**Try These A**

a. A banquet hall charges $15 per person plus a $100 setup fee. The cost for $x$ people is given by the function $f(x) = 100 + 15x$. What are the reasonable domain and range?

b. Eight Ball Billiards charges $5 to rent a table plus $10 per hour of game play, rounded to the nearest whole hour. The cost of playing billiards for $x$ hours is given by the function $f(x) = 5 + 10x$. What are the reasonable domain and range?

2. **Reason quantitatively.** Are the domain and range of $f(x) = 3 + 2x$ that you found in Item 1 the same as the reasonable domain and range of $f(x) = 3 + 2x$ found in Example A? Explain.
3. The graph below represents a real-world situation.

![Graph Image]

a. Identify the domain and range.

b. Describe a real-world situation that matches the graph. Your answers to Part (a) should be the reasonable domain and range for your situation.

c. Identify the independent and dependent variables in your real-world situation.

Check Your Understanding

4. For a function that models a real-world situation, the dependent variable \( y \) represents a person's height. What is a reasonable range? Explain.

5. A tour company charges $25 to hire a tour director plus $75 per tour member. The total cost for a group of \( x \) people is given by \( f(x) = 25 + 75x \). What is the reasonable domain? Explain.

**LESSON 6-3 PRACTICE**

Talk the Talk Cellular charges a base rate of $20 per month for unlimited texts plus $0.15/minute of talk time. The monthly cost for \( x \) minutes is given by \( f(x) = 20 + 0.15x \).

6. **Make sense of problems.** What is the independent variable and what is the dependent variable? Explain how you know.

7. What are the reasonable domain and range? Explain.
Interpreting Graphs of Functions
Shake, Rattle, and Roll

ACTIVITY 6 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 6-1
Use the graph below for Items 1–5.

1. Which point corresponds to the absolute maximum of the function?
   A. B
   B. D
   C. G
   D. H

2. Which represents the range of the function shown in the graph?
   A. \(0 \leq x \leq 10\)
   B. \(1 \leq x \leq 10\)
   C. \(0 \leq y \leq 10\)
   D. \(1 \leq y \leq 10\)

3. Which point does not correspond to a relative minimum?
   A. B
   B. C
   C. E
   D. I

4. Is the function represented by the graph discrete or continuous? Explain.

5. What is the \(y\)-intercept of the function shown in the graph?

6. a. Give the domain and range for the function graphed below. Explain why this graph represents a function.

   ![Graph of a continuous function]

   b. What is the \(y\)-intercept of the function shown in the graph?
   c. Identify any extrema of the function shown in the graph.

Jeff walks at an average rate of 125 yards per minute. Mark’s house is located 2000 yards from Jeff’s house. The graph below shows how far Jeff still needs to walk to reach Mark’s house. Use the graph for Items 7–10.

7. Identify the independent and dependent variables.

8. Identify the absolute minimum and absolute maximum values. What do these values represent?

9. Identify any relative maximums or minimums.

10. What is the \(y\)-intercept? What does it represent?
Lesson 6-2

Use the graph for Items 11–13.

11. What is the domain of the function shown in the graph?

12. What is the range of the function shown in the graph?

13. What is the $y$-intercept of the function shown in the graph?

Use the graph below for Items 14–16.

14. What is the $y$-intercept of the function shown in the graph?

15. Identify any relative maximums.

16. Identify any relative minimums.

Lesson 6-3

A fundraising organization will donate $250 plus half of the money it raises from a charity event. Use this information for Items 17–20.

17. What is the independent variable?

18. What is the dependent variable?

19. What is the reasonable domain? Explain.

20. What is the reasonable range? Explain.

21. Describe a real-world situation that matches the graph shown.

MATHEMATICAL PRACTICES

Look For and Make Use of Structure

22. The graph of a function is a horizontal line. What is true about the absolute maximum and absolute minimum values of this function? Explain.
Graphs of Functions
Experiment Experiences
Lesson 7-1 The Spring Experiment

Learning Targets:
• Graph a function given a table.
• Write an equation for a function given a table or graph.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Look for a Pattern, Sharing and Responding, Think-Pair-Share, Create Representations, Construct an Argument

For the following experiment, you will need a paper cup, a rubber band, a paper clip, a measuring tape, and several washers.

A. Punch a small hole in the side of the paper cup, near the top rim.
B. Use the bent paper clip to attach the paper cup to the rubber band as shown in the diagram in the My Notes section.

1. What is the length of the rubber band?

Drop washers one at a time into the cup. Each time you add a washer, measure the length of the rubber band. Subtract the original length you recorded in Item 1 to find the distance that the rubber band has stretched.

2. Make a table of your data.

<table>
<thead>
<tr>
<th>Number of Washers $x$</th>
<th>Length of Stretch from Original Length $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

3. What patterns do you notice that might help you determine the relationship between the number of washers in the cup and the length of the rubber band stretch?
Lesson 7-1
The Spring Experiment

4. Use your table to make a graph. Be sure to label an appropriate scale and the units on the $y$-axis.

5. Describe your graph.

6. **Model with mathematics.** Use your graph and any patterns you described in Item 3 to write an equation that describes the relationship between the number of washers and the length of the stretch.

7. Use your graph or your equation to predict the length of the stretch for 8 washers and for 10 washers.

A group of students performed a similar experiment with a spring and various masses. The data they collected is shown in the table below.

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>Spring Stretch (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>36</td>
</tr>
</tbody>
</table>

8. Make a graph of the data in the table.
9. **Reason quantitatively.** How much does the spring stretch for each additional gram of mass added? Explain how you found your answer.

10. **Reason abstractly.** Use the students’ data to write an equation that gives the distance $d$ that the spring will stretch in terms of the mass $m$. Explain your equation.

11. Use the equation or the graph to determine the length of the stretch for a mass of 1 gram. Graph the outcome on your graph.

12. Use the equation or the graph to determine the length of the stretch for a mass of 7 grams. Graph the outcome on your graph.

13. Use the equation or the graph to determine the length of the stretch for a mass of 13 grams. Graph the outcome on your graph.

14. a. What do you notice about the points you graphed in Items 11–13?

   b. How could you represent the set of all possible masses and corresponding stretches?

15. What is the $y$-intercept of the graph? What does it represent?

16. What is the reasonable domain? Explain.
Mr. Hardiff’s class conducts an experiment with a spring and a set of weights. They record their data, but some of the information is missing.

<table>
<thead>
<tr>
<th>Weight (oz)</th>
<th>Spring Stretch (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>12.5</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

17. How much does the spring stretch for each additional ounce of weight?

18. Describe how to use your answer to Item 17 to write an equation for the data in the table.

19. Use your equation from Item 18 to complete the table.

Check Your Understanding

20. A 4.5-pound weight stretches a spring 18 inches and a 7.5-pound weight stretches the same spring 30 inches. How much does the spring stretch for each additional pound of weight? Explain how you found your answer.

LESSON 7-1 PRACTICE

Jeremy and his classmates conduct an experiment with a set of weights and a spring. They record their results in the table. Use the table to answer Items 21–24.

<table>
<thead>
<tr>
<th>Student</th>
<th>Mass (lb)</th>
<th>Spring Stretch (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeremy</td>
<td>5</td>
<td>7.5</td>
</tr>
<tr>
<td>Adele</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Roberto</td>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>Shanice</td>
<td>21</td>
<td>36</td>
</tr>
<tr>
<td>Guillaume</td>
<td>28</td>
<td>42</td>
</tr>
</tbody>
</table>

21. Make a graph of the data.

22. Critique the reasoning of others. Which student made a mistake when taking their turn at the experiment? Explain how you know.

23. If the mistake in Item 22 were corrected, what would the correct data point be?

24. Write an equation to describe the students’ data, using the corrected data point you identified in Item 23.
Lesson 7-2
The Falling Object Experiment

Learning Target:

- Graph a function describing a real-world situation and identify and interpret key features of the graph.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Look for a Pattern, Construct an Argument, Think-Pair-Share, Summarizing, Sharing and Responding

1. The Empire State Building in New York City is 1454 feet tall. How long do you think it will take a penny dropped from the top of the Empire State Building to hit the ground?

In 1589, the mathematician and scientist Galileo conducted an experiment to answer a question much like the one in Item 1. Galileo dropped balls from the top of the Leaning Tower of Pisa in Italy and determined the time it took them to reach the ground. Galileo used several balls identical in shape but differing in mass. Because the balls all reached the ground in the same amount of time, he developed the theory that all objects fall at the same rate.

Galileo's findings can be represented with the equation $h(t) = 1600 - 16t^2$, where $h(t)$ represents the height in feet of an object $t$ seconds after it has been dropped from a height of 1600 feet.

2. Make a table of values for Galileo's function $h(t) = 1600 - 16t^2$.

<table>
<thead>
<tr>
<th>$t$ (seconds)</th>
<th>$h(t)$ (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
3. **Construct viable arguments.** Why would negative domain values not be appropriate in this context?

4. Using your table of values, graph Galileo’s function.

5. What is the reasonable domain of the function represented in your graph? What is the reasonable range?

6. What is the \( y \)-intercept?

7. What does the \( y \)-intercept represent?

8. What is the \( x \)-intercept? What does the \( x \)-intercept represent?

9. Identify any extrema of the function shown in the graph. What do the extrema represent?

“Your homework assignment is to graph this function,” your math teacher says. She then points to the following function on the board:

\[ f(x) = x^2 - 2x \]

In this case, the function is not limited by a real-world situation. Therefore, it is important to use different types of domain values as you prepare to graph.
10. Using various values for $x$, make a table of values for $f(x) = x^2 - 2x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11. Using your table of values, graph the function.

12. Describe the differences between the domain of $f(x) = x^2 - 2x$ and the domain of Galileo's function.

13. State the range of $f(x) = x^2 - 2x$.

14. Identify the $y$-intercept of $f(x) = x^2 - 2x$.

15. What is the absolute maximum of $f(x) = x^2 - 2x$? What is the absolute minimum?
LESSON 7-2 PRACTICE

The area of a rectangle with a perimeter of 20 units is given by \( f(w) = 10w - w^2 \), where \( w \) is the width of the rectangle. Assume that \( w \) is a whole number. Use this function to answer Items 17–20.

17. Make a table of values and a graph of the function.

18. Attend to precision. Give a reasonable domain for the function in this context. Explain your answers.

19. Identify the \( y \)-intercept of the function. What does the \( y \)-intercept represent within this context?

20. What is the absolute maximum of the function? What is the absolute minimum?

For Items 21–23, use the function \( f(x) = x^2 - 9 \).

21. Make a table of values and a graph of the function.

22. What are the domain and range?

23. Identify the \( y \)-intercept, the absolute maximum, and the absolute minimum.
Learning Targets:

- Given a verbal description of a function, make a table and a graph of the function.
- Graph a function and identify and interpret key features of the graph.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Look for a Pattern, Construct an Argument, Paraphrasing, Marking the Text, Think-Pair-Share

In the late nineteenth century, the scientist Marie Curie performed experiments that led to the discovery of radioactive substances.

A radioactive substance is a substance that gives off radiation as it decays. Scientists describe the rate at which a radioactive substance decays as its half-life. The half-life of a substance is the amount of time it takes for one-half of the substance to decay.

1. Radium has a half-life of 1600 years. How much radium will be left from a 1000-gram sample after 1600 years?

2. How much radium will be left after another 1600 years?

3. Suppose a radioactive substance has a half-life of 1 second and you begin with a sample of 4 grams. Complete the table of values.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Amount Remaining (grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
4. Graph the data from the table on the grid below.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Amount Remaining (grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

5. Make use of structure. Will the amount of the substance that remains ever reach 0? Explain.

6. What are the reasonable domain and range of the function represented in the graph? Explain.

7. What is the y-intercept and what does it represent?

8. Identify the absolute maximum and minimum of the function represented in the graph, and tell what they represent in the context.
The function that describes the substance's decay is \( f(x) = 4\left(\frac{1}{2}\right)^x \). The graph of this function when it does not model a real-world situation is shown below.

9. What are the domain and range of the function?

10. How is this graph different from your graph in Item 4?

11. How do the values of \( y \) change as the values of \( x \) increase?

12. How do the values of \( y \) change as the values of \( x \) decrease?

13. Identify the absolute maximum and absolute minimum of the function.
Check Your Understanding

14. A scientist has $g$ grams of a radioactive substance. Write an expression that shows the amount of the substance that remains after one half-life.

15. **Critique the reasoning of others.** Dylan looked at the function $f(x) = 4\left(\frac{1}{2}\right)^x$ and said, “This function is always greater than 0, so 0 is the absolute minimum.” Explain why Dylan is incorrect.

**LESSON 7-3 PRACTICE**

Suppose the value of your new car is reduced by half every year that you own it. You paid $20,000 for your new car.

16. Describe how this situation is similar to the half-life of a radioactive substance.

17. Copy and complete the table below.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20,000</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

18. **Make sense of problems.** For insurance purposes, a vehicle is considered scrap when its value falls below $500. After how many years will your new car be considered scrap?
ACTIVITY 7 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 7-1
A weight of 15 ounces stretches a spring 10 inches. A weight of 24 ounces stretches the same spring 16 inches. Use this information to answer Items 1–4.

1. How many inches does the spring stretch per ounce of additional weight?
   A. \( \frac{2}{3} \) inch
   B. \( \frac{3}{2} \) inches
   C. 25 inches
   D. 150 inches

2. Write an equation to describe the relationship between the distance \( d \) that the spring stretches and the weight \( w \) that is attached to it.

3. How much will the spring stretch for a weight of 9 ounces?

4. The spring is stretched 14 inches. How many ounces is the weight that is attached to it?

A spring stretches 2.5 inches for each ounce of weight. Use this information for Items 5–7.

5. Determine a function that represents this situation.

6. If you were to graph the function represented by this situation, what would be the reasonable domain? Explain.

7. Which of the following data points would not lie on the graph representing this function?
   A. (0, 0)
   B. (1, 2.5)
   C. (2.5, 1)
   D. (10, 25)

Lesson 7-2
Suppose that the height of an object after \( x \) seconds is given by \( f(x) = 100 - 4x^2 \), as shown in the graph below.

Use the function or the graph for Items 8–14.

8. What is the reasonable domain of the function?

9. What is the reasonable range of the function?

10. Identify the \( y \)-intercept of the function.

11. What does the \( y \)-intercept represent?

12. Identify the \( x \)-intercept of the function.

13. What does the \( x \)-intercept represent?

14. Loni says that because of the negative sign in front of \( 4x^2 \), the reasonable domain for this function is only negative values. Is her reasoning correct? Explain.
Lesson 7-3

15. The half-life of a radioactive substance is 1 hour. If you begin with 100 ounces of the substance, how many hours does it take for 12.25 ounces to remain?

The graph below represents a radioactive decay situation. Use this graph for Items 16–18.

16. What is the original amount of the radioactive substance? Explain how you know.

17. What are the reasonable domain and range?

18. Identify the absolute maximum and absolute minimum values of the function. What do these values represent?

Barry has a piece of paper whose area is 150 square inches. He cuts the paper in half and discards one of the pieces. He repeats this procedure several times. Use this information for Items 19–24.

19. Copy and complete the table below to show the area of the remaining piece of paper after \(x\) cuts.

<table>
<thead>
<tr>
<th>Number of Cuts, (x)</th>
<th>Area of Remaining Piece, (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
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<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

20. Describe how this situation is similar to the half-life of a radioactive substance.

21. If you were to graph the points from the table, would you connect the points? Explain.

22. Describe how the reasonable domain in this situation is different from the reasonable domain in a radioactive decay situation.

23. Identify the \(y\)-intercept. What does it represent?

24. Identify the absolute maximum value. What does it represent?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

25. Maude receives $100 for her birthday. “I am going to spend half of my birthday money each day until none is left,” she decides. Is it reasonable for her to believe that she will eventually spend all of the money? Justify your answer.
Transformations of Functions
Transformers
Lesson 8-1 Exploring \( f(x) + k \)

Learning Targets:
- Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \).
- Identify the transformation used to produce one graph from another.

**SUGGESTED LEARNING STRATEGIES:** Look for a Pattern, Interactive Word Wall, Think-Pair-Share, Create Representations, Discussion Groups

The equation and the graph of \( y = x \) or \( f(x) = x \) are referred to as the linear **parent function.** The graph of \( f(x) = x \) is shown below.

1. Complete the table for \( g(x) = x + 5 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = x )</th>
<th>( g(x) = x + 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-3</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>-2</td>
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</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

2. **Make use of structure.** How do the \( y \)-values for \( g(x) \) compare to the \( y \)-values for \( f(x) \)? Make a conjecture about the graph of \( g(x) \). As you share your ideas with your group, be sure to use mathematical terms and academic vocabulary precisely. Make notes to help you remember the meaning of new words and how they are used to describe mathematical concepts.

**MATH TERMS**

A **parent function** is the most basic function of a particular category or type.
Lesson 8-1
Exploring \( f(x) + k \)

3. Test your conjecture by using a graphing calculator to graph \( g(x) = x + 5 \). Graph this on the grid in Item 1.
   a. What is the \( y \)-intercept of the parent function?
   
   b. What is the \( y \)-intercept of \( g(x) \)?
   
   c. What is the \( x \)-intercept of the parent function? What is the zero of the function \( f(x) \)?
   
   d. What is the \( x \)-intercept of \( g(x) \)? What is the zero of the function?
   
   e. Revisit your original conjecture in Item 2 and revise it if necessary. How does the graph of \( g(x) \) differ from the graph of the parent function, \( f(x) = x \)?

The graph of \( f(x) = x^3 \) is shown below.

![Graph of \( f(x) = x^3 \)](image)

4. Make a conjecture about the graph of \( g(x) = x^3 - 4 \).

5. Graph both \( f(x) \) and \( g(x) \) on a graphing calculator. Sketch the graph of \( g(x) \) on the grid above. Label a few points on each graph.

6. Revisit your original conjecture in Item 4 about the graph of \( g(x) \) and revise it if necessary. How does the graph of \( g(x) \) differ from the graph of \( f(x) \)?

7. **Express regularity in repeated reasoning.** How does the value of \( k \) in the equation \( g(x) = f(x) + k \) change the graph of \( f(x) \)?
Exploring $f(x) + k$

A change in the position, size, or shape of a graph is a **transformation**. The changes to the graphs in Items 1–6 are examples of a transformation called a **vertical translation**.

8. In the figure, the graphs of $g(x)$ and $h(x)$ are vertical translations of the graph of $f(x) = 2^x$.
   a. Write the equation for $g(x)$.
   b. Write the equation for $h(x)$.

Check Your Understanding

9. Without graphing, describe the transformation from the graph of $f(x) = x^2$ to the graph of $g(x) = x^2 + 7$.
10. Suppose $f(x) = x - 2$. Describe the transformation from the graph of $f(x)$ to the graph of $g(x) = x + 3$. Use a graphing calculator to check your answer.

Ray’s Gym charges an initial sign-up fee of $25.00 and a monthly fee of $15.00.

11. **Reason abstractly.** Write a function that describes the gym’s total membership fee for $x$ months.

12. Graph the function you wrote in Item 11 on the grid below. Label several points on the graph.

13. Identify the $y$-intercept. What does the $y$-intercept represent?
14. How would the function change if the initial sign-up fee were increased by $5.00? How would the graph change?

Check Your Understanding

15. The membership fee at Gina's Gym is given by the function $g(x) = 15x + 20$, where $x$ is the number of months.
   a. How do the fees at Gina's Gym compare to those at Ray's Gym?
   b. Without graphing, describe how the graph of $g(x)$ compares to the graph of $f(x)$.

16. The $y$-intercept of a function $f(x)$ is $(0, b)$. What is the $y$-intercept of $f(x) + k$?

Lesson 8-1 Practice

Identify the transformation from the graph of $f(x) = x^2$ to the graph of $g(x)$. Then graph $f(x)$ and $g(x)$ on the same coordinate plane.

17. $g(x) = x^2 - 7$
18. $g(x) = x^2 + 10$

Write the equation of the function described by each of the following transformations of the graph of $f(x) = x^2$.

19. Translated up 9 units
20. Translated down 5 units

Each graph shows a vertical translation of the graph of $f(x) = x$. Write an equation to describe each graph.

21.
22.

23. Model with mathematics. Orange Taxi charges $2.75 as soon as you step into the taxi and $2.50 per mile. Magenta Taxi charges $3.25 as soon as you step into the taxi and $2.50 per mile.
   a. Write a function $f(x)$ that describes the total cost of a ride of $x$ miles with Orange Taxi. Write a function $g(x)$ that describes the total cost of a ride of $x$ miles with Magenta Taxi.
   b. Without graphing, explain how the graph of $g(x)$ compares to the graph of $f(x)$.
   c. Check your answer to Part (b) by graphing the functions.
Learning Targets:
• Identify the effect on the graph of replacing \( f(x) \) by \( f(x + k) \).
• Identify the transformation used to produce one graph from another.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Look for a Pattern, Create Representations, Think-Pair-Share, Discussion Groups

The function \( f(x) = |x| \) is graphed below.

1. Write a new function, \( g(x) \), by replacing \( x \) with \( x + 7 \).

2. Graph both \( f(x) = |x| \) and \( g(x) \) on a graphing calculator. Sketch the graph of \( g(x) \) on the grid above, labeling at least a few points on each graph.

3. What is the \( x \)-intercept of \( f(x) = |x| \)?

4. What is the \( x \)-intercept of \( g(x) \)?

5. Describe the transformation from the graph of \( f(x) = |x| \) to the graph of \( g(x) \).

Note that the function \( g(x) \) can be written as \( f(x + 7) \). This means that \( x \) is replaced with \( x + 7 \) in the function \( f(x) \).

MATH TERMS
An absolute value function is written as \( f(x) = |x| \) and is defined by

\[
f(x) = \begin{cases} 
-x & \text{if } x < 0 \\
x & \text{if } x \geq 0 
\end{cases}
\]

CONTACT AP
The vertex of an absolute value function is an example of a cusp in a graph. A graph has a cusp at a point where there is an abrupt change in direction.
The graph of \( f(x) = x^3 \) is shown below.

6. Make a conjecture about the graph of \( g(x) = (x - 3)^3 \).

7. Graph both \( f(x) \) and \( g(x) \) on a graphing calculator. Sketch the graph of \( g(x) \) on the grid above, labeling at least a few points on each graph.

8. Revisit your original conjecture in Item 6 about the graph of \( g(x) \) and revise it if necessary. How does the graph of \( g(x) \) differ from the graph of \( f(x) \)?

9. How does the value of \( k \) in the equation \( g(x) = f(x + k) \) change the graph of the function \( f(x) \)?

The changes to the graphs in Items 1–8 are examples of a transformation called a **horizontal translation**.

10. The figure shows the graph of the function \( f(x) = 2^x \).
   a. Without using a graphing calculator, sketch the graph of \( g(x) = f(x + 8) = 2^{x+8} \) on the grid.
   
   b. Use a graphing calculator to check your graph in Part (a). Revise your graph if necessary.
Lesson 8-2
Exploring \( f(x + k) \)

Check Your Understanding

11. Without graphing, describe the transformation from the graph of \( f(x) = x^2 \) to the graph of \( g(x) \).
   a. \( g(x) = (x + 4)^2 \)
   b. \( g(x) = f(x - 7) \)
   c. \( g(x) = (x - 2)^2 + 5 \)
   d. \( g(x) = (x + 9)^2 - 1 \)

12. The function \( f(x) = x^2 \) and another function, \( g(x) \), are graphed below. Write the equation for \( g(x) \). Explain how you found your answer.

   ![Graph of \( f(x) = x^2 \) and \( g(x) \) graphed below]

13. **Make sense of problems.** Julio went to a theme park in July. He paid $15 to enter the park and $3.00 for each ride. He went on \( x \) rides.
   a. Write a function that describes the total cost of Julio's trip to the theme park.

   b. Julio went back to the theme park in September. The entrance fee was the same and each ride still cost $3.00. However, this time Julio went on 5 more rides. Use your function from Part (a) to describe Julio's second trip.

   c. How does the equation for Julio's second trip to the park change the graph of the first trip?

   d. What kind of transformation describes the change from the first graph to the second graph?

   e. Julio went to the park again in October and went on 8 fewer rides than he did in July. Use your function from Part (a) to describe Julio's third trip. How does this change the initial graph?
f. Julio goes to the park again in November. Now it is the off-season and the entrance fee is $10 less than it was in July. He goes on the same number of rides as he did in July. Write a function to describe Julio’s fourth trip. How does the graph of the initial trip change with this new situation?

**Check Your Understanding**

14. The x-intercept of the function \( f(x) \) is \((a, 0)\). What is the x-intercept of the function \( f(x + k) \)?

15. Without graphing, explain how the graph of \( y = (x - 4)^3 \) is related to the graph of \( y = (x + 4)^3 \).

**LESSON 8-2 PRACTICE**

Identify the transformation from the graph of \( f(x) = x^2 \) to the graph of \( g(x) \). Then graph \( f(x) \) and \( g(x) \) on the same coordinate plane.

16. \( g(x) = (x - 1)^2 \)

17. \( g(x) = (x + 3)^2 \)

Write the equation of the function described by each of the following transformations of the graph of \( f(x) = x^3 \).

18. Translated 7 units to the left

19. Translated 8 units to the right

20. Each graph shows a horizontal translation of the graph of \( f(x) = x \). Write an equation to describe each graph.

   a. 

   b. 

   c. Critique the reasoning of others. Molly said that the graphs above are also vertical translations of the graph of \( f(x) = x \). Is Molly correct? Explain.

21. How does the graph of \( h(x) = |x - 4| \) compare with the graph of \( f(x) = |x| \)?
Activity 8 • Transformations of Functions

ACTIVITY 8 PRACTICE
Write your answers on notebook paper.
Show your work.

Lesson 8-1
In Items 1–4, identify the transformation from the graph of \( f(x) = x^3 \) to the graph of \( g(x) \).
1. \( g(x) = x^3 + 11 \)
2. \( g(x) = x^3 - 4 \)
3. \( g(x) = x^3 + 0.1 \)
4. \( g(x) = -2 + x^3 \)
5. The graph of \( f(x) = x^2 \) is translated 9 units down to create the graph of \( g(x) \). Which of the following is the equation for \( g(x) \)?
   A. \( g(x) = x^2 + 9 \)
   B. \( g(x) = x^2 - 9 \)
   C. \( g(x) = (x + 9)^2 \)
   D. \( g(x) = (x - 9)^2 \)

In Items 6 and 7, each graph shows a vertical translation of the graph of \( f(x) = x \). Write an equation to describe the graph. Identify the zeros of each function.

6.

7.

For Items 8 and 9, determine the equation of the function described by each of the following transformations of the graph of \( f(x) = 3^x \).
8. Translated 15 units down
9. Translated 2.1 units up

10. An air conditioner costs $450 plus $40 per month to operate.
   a. Write a function that describes the total cost of buying and operating the air conditioner for \( x \) months.
   b. Use your calculator to graph the function.
   c. What is the \( y \)-intercept? What does it represent?
   d. How would the function change if the price of the air conditioner were reduced to $425? How would the graph change?

Given that \( g(x) = f(x) + k \), with \( k \neq 0 \), determine whether each statement is always, sometimes, or never true.
11. The graph of \( g(x) \) is a vertical translation of the graph of \( f(x) \).
12. The graphs of \( f(x) \) and \( g(x) \) are both lines.
13. The graph of \( f(x) \) has the same \( y \)-intercept as the graph of \( g(x) \).
14. Caitlin drew the graph of \( f(x) = x^2 \). Then she translated the graph 6 units up to get the graph of \( g(x) \). Next, she translated the graph of \( g(x) \) 8 units down to get the graph of \( h(x) \). Which of these is an equation for \( h(x) \)?
   A. \( h(x) = x^2 + 14 \)
   B. \( h(x) = x^2 + 2 \)
   C. \( h(x) = x^2 - 2 \)
   D. \( h(x) = x^2 - 14 \)
Lesson 8-2

In Items 15–18, identify the transformation from the graph of \( f(x) = 2^x \) to the graph of \( g(x) \).

15. \( g(x) = 2^x - 3 \)
16. \( g(x) = 2^{(x - 3)} \)
17. \( g(x) = 2^x + 4 \)
18. \( g(x) = 2^{(x + 4)} \)

19. The graph of which function is a translation of the graph of \( f(x) = x^2 \) five units to the right?
   A. \( g(x) = x^2 - 5 \)
   B. \( g(x) = (x + 5)^2 \)
   C. \( g(x) = (x - 5)^2 \)
   D. \( g(x) = x^2 + 5 \)

Write the equation of the function described by each of the following transformations of the graph of \( f(x) = x^3 \).

20. Translated 7 units up
21. Translated 4 units down
22. Translated 2 units right
23. Translated 5 units down
24. Translated 3 units left
25. The figure shows the graph of \( f(x) = x^4 \) and the graph of \( g(x) \). Write an equation for the graph of \( g(x) \).

Without graphing, describe the transformation from the graph of \( f(x) = x^2 \) to the graph of \( g(x) \).

26. \( g(x) = (x - 7)^2 + 1 \)
27. \( g(x) = f(x + 4) \)
28. \( g(x) = (x + 9)^2 - 0.2 \)
29. \( g(x) = f(x - 2) - 3 \)

30. The graph of \( f(x) \) is shown below. Which of the following is a true statement about the graph of \( g(x) = f(x + 3) \)?

   A. The \( x \)-intercept of \( g(x) \) is (3, 0).
   B. The \( x \)-intercept of \( g(x) \) is (−3, 0).
   C. The \( y \)-intercept of \( g(x) \) is (0, 3).
   D. The \( y \)-intercept of \( g(x) \) is (0, −3).

MATHEMATICAL PRACTICES

Model with Mathematics

31. In 2011, the ticket price for entrance to a state fair was $12. Each ride had an additional $4.00 fee. In 2012, the entrance ticket cost $15 and the rides remained $4.00 each.
   a. Write a function \( f(x) \) for the cost of visiting the fair and riding \( x \) rides in 2011.
   b. Write a function \( g(x) \) for the cost of visiting the fair and riding \( x \) rides in 2012.
   c. What transformation could you use to obtain the graph of \( g(x) \) from the graph of \( f(x) \)?
   d. What transformation could you use to obtain the graph of \( f(x) \) from the graph of \( g(x) \)?
While on vacation, Jorge and Jackie traveled to Bryce Canyon National Park in Utah. They were impressed by the differing elevations at the viewpoints along the road. The graph describes the elevations for several viewpoints in terms of the time since they entered the park.

1. The graph represents a function $E(t)$. Describe why the graph represents a function. Identify the domain and range of the function.
2. Is this discrete or continuous data? Explain.
3. What is the $y$-intercept? Interpret the meaning of the $y$-intercept in the context of the problem.
4. Identify a relative maximum of the function represented by the graph.
5. What is the absolute maximum of the function represented by the graph? What does it represent?
6. Identify a relative minimum of the function represented by the graph.
7. What is the absolute minimum of the function represented by the graph? What does it represent?

While at Bryce Canyon National Park, Jorge and Jackie hiked at an average speed of about 2 miles per hour.

8. Copy and complete the table below to show the distance hiked by a person whose constant speed is 2 miles per hour.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Distance (miles)</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
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<tr>
<td>2</td>
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<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

9. Write a function $f(x)$ to describe the data in the table. What are the reasonable domain and range?

10. Create a graph of the function.

11. How long will it take this person to hike 5 miles? Justify your answer.

12. On the same coordinate grid that you used in Item 9, create a graph of another function by translating the graph 5 units up.

13. Write a function to describe the graph you created in Item 12. Explain how you determined your answer.
## Scoring Guide

The solution demonstrates the following characteristics:

<table>
<thead>
<tr>
<th>Mathematics Knowledge and Thinking (Items 1, 3–7)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear and accurate identification of key features of the function and its graph, including domain, range, ( y )-intercept, maximums, and minimums</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Correct identification of most of the key features of the function and its graph, including domain, range, ( y )-intercept, maximums, and minimums</td>
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<td>•</td>
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</tr>
<tr>
<td>Partially correct identification of some of the key features of the function and its graph, including domain, range, ( y )-intercept, maximums, and minimums</td>
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<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Inaccurate or incomplete identification of key features of the function and its graph, including domain, range, ( y )-intercept, maximums, and minimums</td>
<td>•</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving (Item 11)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appropriate and efficient strategy that results in a correct answer</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Strategy that may include unnecessary steps but results in a correct answer</td>
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<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Strategy that results in some incorrect answers</td>
<td>•</td>
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<td>•</td>
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</tr>
<tr>
<td>No clear strategy when solving problems</td>
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<table>
<thead>
<tr>
<th>Mathematical Modeling / Representations (Items 8–10, 12, 13)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective understanding of how to complete a table of real-world data, and how to write, graph, and interpret the associated function</td>
<td>•</td>
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<td>•</td>
</tr>
<tr>
<td>Fluency in translating a graph and writing the associated function</td>
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<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Largely correct understanding of how to complete a table of real-world data, and how to write, graph, and interpret the associated function</td>
<td>•</td>
<td>•</td>
<td>•</td>
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</tr>
<tr>
<td>Little difficulty translating a graph and writing the associated function</td>
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<td>•</td>
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</tr>
<tr>
<td>Partial understanding of how to complete a table of real-world data, and how to write, graph, and interpret the associated function</td>
<td>•</td>
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<td>•</td>
</tr>
<tr>
<td>Some difficulty translating a graph and writing the associated function</td>
<td>•</td>
<td>•</td>
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<td>•</td>
</tr>
<tr>
<td>Inaccurate or incomplete understanding of how to complete a table of real-world data, and how to write, graph, and interpret the associated function</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Significant difficulty translating a graph and writing the associated function</td>
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<table>
<thead>
<tr>
<th>Reasoning and Communication (Items 1–3, 5, 7, 13)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
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</thead>
<tbody>
<tr>
<td>Precise use of appropriate math terms and language to describe key features of a graph and to explain how a function rule was determined from a translated graph</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Clear and accurate interpretations of the graph of a function</td>
<td>•</td>
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</tr>
<tr>
<td>Adequate description of key features of a graph</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Reasonable interpretations of the graph of a function</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Adequate explanation of how a function rule was determined from a translated graph</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Confusing description of key features of a graph</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Partially correct interpretations of the graph of a function</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Confusing explanation of how a function was determined from a translated graph</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Incomplete or inaccurate description of key features of a graph</td>
<td>•</td>
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</tr>
<tr>
<td>Incomplete or inaccurate interpretation of the graph of a function</td>
<td>•</td>
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<td>•</td>
</tr>
<tr>
<td>Incomplete or inaccurate explanation of how a function was determined from a translated graph</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
</tbody>
</table>
Rates of Change
Ramp it Up
Lesson 9-1 Slope

Learning Targets:
• Determine the slope of a line from a graph.
• Develop and use the formula for slope.

SUGGESTED LEARNING STRATEGIES: Close Reading, Summarizing, Sharing and Responding, Discussion Groups, Construct an Argument, Identify a Subtask

Margo’s grandparents are moving in with her family. The family needs to make it easier for her grandparents to get in and out of the house. Margo has researched the specifications for building stairs and wheelchair ramps. She found the government website that gives the Americans with Disabilities Act (ADA) accessibility guidelines for wheelchair ramps and discovered the following diagram:

Then, Margo decided to look for the requirements for building stairs and found the following diagram:

Review with your group the background information that is given as you solve the following items.

1. What do you think is meant by the terms *rise* and *run* in this context?

<table>
<thead>
<tr>
<th>Slope</th>
<th>Maximum Rise</th>
<th>Maximum Run</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in.</td>
<td>mm</td>
</tr>
<tr>
<td>$\frac{1}{16} \leq m \leq \frac{1}{12}$</td>
<td>30, 760</td>
<td>30, 9</td>
</tr>
<tr>
<td>$\frac{1}{20} \leq m &lt; \frac{1}{16}$</td>
<td>30, 760</td>
<td>40, 12</td>
</tr>
</tbody>
</table>
Consider the line in the graph below:

Vertical change can be represented as a \textit{change in} \(y\), and horizontal change can be represented by a \textit{change in} \(x\).

2. What is the vertical change between:
   a. points \(A\) and \(B\)?
   b. points \(A\) and \(C\)?
   c. points \(C\) and \(D\)?

3. What is the horizontal change between:
   a. points \(A\) and \(B\)?
   b. points \(A\) and \(C\)?
   c. points \(C\) and \(D\)?

The ratio of the vertical change to the horizontal change determines the \textit{slope} of the line.

\[
\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in} \ y}{\text{change in} \ x} = \frac{\Delta y}{\Delta x}
\]

4. Find the slope of the segment of the line connecting:
   a. points \(A\) and \(B\)
   b. points \(A\) and \(C\)
   c. points \(C\) and \(D\)

5. What do you notice about the slope of the line in Items 4a, 4b, and 4c?

6. What does your answer to Item 5 indicate about points on a line?
7. Slope is sometimes referred to as \( \frac{\text{rise}}{\text{run}} \). Explain how the ratio \( \frac{\text{rise}}{\text{run}} \) relates to the ratios for finding slope mentioned above.

8. **Reason quantitatively.** Would the slope change if you counted the run (horizontal change) before you counted the rise (vertical change)? Explain your reasoning.

9. Determine the slope of the line graphed below.

```
<table>
<thead>
<tr>
<th>y</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-10</td>
</tr>
<tr>
<td>8</td>
<td>-8</td>
</tr>
<tr>
<td>6</td>
<td>-6</td>
</tr>
<tr>
<td>4</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-4</td>
<td>4</td>
</tr>
<tr>
<td>-6</td>
<td>6</td>
</tr>
<tr>
<td>-8</td>
<td>8</td>
</tr>
<tr>
<td>-10</td>
<td>10</td>
</tr>
</tbody>
</table>
```

![Graph of a line with points at (-10,10), (-8,8), (-6,6), (-4,4), (-2,2), (2,4), (4,6), (6,8), (8,10), (10,10).]
Although the slope of a line can be calculated by looking at a graph and counting the vertical and horizontal change, it can also be calculated numerically.

10. Recall that the slope of a line is the ratio \( \frac{\text{change in } y}{\text{change in } x} \).

a. Identify two points on the graph above and record the coordinates of the two points that you selected.

<table>
<thead>
<tr>
<th>x-coordinate</th>
<th>y-coordinate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st point</td>
<td></td>
</tr>
<tr>
<td>2nd point</td>
<td></td>
</tr>
</tbody>
</table>

b. Which coordinates relate to the vertical change on the graph?

c. Which coordinates relate to the horizontal change on a graph?

d. Determine the vertical change.

e. Determine the horizontal change.

f. Calculate the slope of the line. How does this slope compare to the slope that you found in Item 9?

g. If other students in your class selected different points for this problem, should they have found different values for the slope of this line? Explain.

11. It is customary to label the coordinates of the first point \((x_1, y_1)\) and the coordinates of the second point \((x_2, y_2)\).

a. Write an expression to calculate the vertical change, \( \Delta y \), of the line through these two points.

b. Write an expression to calculate the horizontal change, \( \Delta x \), of the line through these two points.

c. Write an expression to calculate the slope of the line through these two points.
Lesson 9-1
Slope

12. Use the slope formula to determine the slope of a line that passes through the points (4, 9) and (−8, −6).
13. Use the slope formula to determine the slope of the line that passes through the points (−5, −3) and (9, −10).
14. Explain how to find the slope of a line from a graph.
15. Explain how to find the slope of a line when given two points on the line.

LESSON 9-1 PRACTICE

16. Find \( \triangle x \) and \( \triangle y \) for the points (7, −2) and (9, −7).
17. Critique the reasoning of others. Connor determines the slope between (−2, 4) and (3, −3) by calculating \( \frac{4 - (-3)}{-2 - 3} \). April determines the slope by calculating \( \frac{3 - (-2)}{-3 - 4} \). Explain whose reasoning is correct.
18. When given a table of ordered pairs, you can find the slope by choosing any two ordered pairs from the table. Determine the slope represented in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>−1</td>
</tr>
</tbody>
</table>

19. Determine the slope of the given line.

[Diagram of a line with labeled axes]
Learning Targets:

- Calculate and interpret the rate of change for a function.
- Understand the connection between rate of change and slope.

**SUGGESTED LEARNING STRATEGIES:** Discussion Groups, Create Representations, Look for a Pattern, Think-Pair-Share

The rate of change for a function is the ratio of the change in $y$, the dependent variable, to the change in $x$, the independent variable.

1. Margo went to the lumberyard to buy supplies to build the wheelchair ramp. She knows that she will need several pieces of wood. Each piece of wood costs $3.
   a. **Model with mathematics.** Write a function $f(x)$ for the total cost of the wood pieces if Margo buys $x$ pieces of wood.
   b. Make an input/output table of ordered pairs and then graph the function.

<table>
<thead>
<tr>
<th>Pieces of Wood, $x$</th>
<th>Total Cost, $f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c. What is the slope of the line that you graphed?

   d. By how much does the cost increase for each additional piece of wood purchased?
Lesson 9-2
Slope and Rate of Change

e. How does the slope of this line relate to the situation with the pieces of wood?

f. Is there a relationship between the slope of the line and the equation of the line? If so, describe that relationship.

2. Margo is going to work with a local carpenter during the summer. Each week she will earn $10.00 plus $2.00 per hour.
   a. Write a function \( f(x) \) for Margo’s total earnings if she works \( x \) hours in one week.

   b. Make an input/output table of ordered pairs and then graph the function. Label your axes.

<table>
<thead>
<tr>
<th>Hours, ( x )</th>
<th>Earnings, ( f(x) ) (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
c. How much will Margo's earnings change if she works 6 hours instead of 2? If she works 4 hours instead of 3? How much do Margo's earnings change for each additional hour worked?

d. Does the function have a constant rate of change? If so, what is it?

e. What is the slope of the line that you graphed?

f. Describe the meaning of the slope within the context of Margo's job.

g. Describe the relationship between the slope of the line, the rate of change, and the equation of the line.

h. How much will Margo earn if she works for 8 hours in one week?

3. By the end of the summer, Margo has saved $375. Recall that each of the small pieces of wood costs $3.
   a. Write a function \( f(x) \) for the amount of money that Margo still has if she buys \( x \) pieces of wood.
Lesson 9-2
Slope and Rate of Change

b. Make an input/output table of ordered pairs and then graph the function.

<table>
<thead>
<tr>
<th>Pieces of Wood, x</th>
<th>Money Remaining, f(x) (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Margo's Savings

f(x)

Money Remaining (in dollars)

<table>
<thead>
<tr>
<th>Money Remaining (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
</tr>
<tr>
<td>450</td>
</tr>
<tr>
<td>400</td>
</tr>
<tr>
<td>350</td>
</tr>
<tr>
<td>300</td>
</tr>
<tr>
<td>250</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>150</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

c. How much will the amount Margo has saved change if she buys 100 instead of 25 pieces of wood? If she buys 50 instead of 0 pieces of wood? For each additional piece of wood? Explain.

d. Does the function have a constant rate of change? If so, what is it?

e. What is the slope of the line that you graphed?

f. How are the rate of change of the function and the slope related?

g. Describe the meaning of the slope within the context of Margo's savings.

h. How does this slope differ from the other slopes that you have seen in this activity?
Check Your Understanding

4. The constant rate of change of a function is $-5$. Describe the graph of the function as you look at it from left to right.

5. Does the table represent data with a constant rate of change? Justify your answer.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>11</td>
<td>40</td>
</tr>
</tbody>
</table>

LESSON 9-2 PRACTICE

6. The art museum charges an initial membership fee of $50.00. For each visit the museum charges $15.00.
   a. Write a function $f(x)$ for the total amount charged for $x$ trips to the museum.
   b. Make a table of ordered pairs and then graph the function.
   c. What is the rate of change? What is the slope of the line?
   d. How does the slope of this line relate to the number of museum visits?

7. Critique the reasoning of others. Simone claims that the slope of the line through $(-2, 7)$ and $(3, 0)$ is the same as the slope of the line through $(2, 1)$ and $(12, -13)$. Prove or disprove Simone’s claim.
Learning Targets:
- Show that a linear function has a constant rate of change.
- Understand when the slope of a line is positive, negative, zero, or undefined.
- Identify functions that do not have a constant rate of change and understand that these functions are not linear.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Think-Pair-Share, Construct an Argument, Sharing and Responding, Summarizing

You have seen that for a linear function, the rate of change is constant and equal to the slope of the line. This is because linear functions increase or decrease by equal differences over equal intervals. Look at the graph below.

1. Over the interval 2 to 4, by how much does the function increase? Explain.

2. Over the equal interval 8 to 10, by how much does the function increase? Explain.

“Equal differences over equal intervals” is an equivalent way of referring to constant slope. “Differences” refers to $\Delta y$, and “intervals” refers to $\Delta x$. “Equal differences over equal intervals” means $\frac{\Delta y}{\Delta x}$, which represents the slope, will always be the same.
3. The table below represents a function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>62</td>
</tr>
<tr>
<td>-6</td>
<td>34</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>7</td>
<td>47</td>
</tr>
</tbody>
</table>

a. Determine the rate of change between the points $(-8, 62)$ and $(-6, 34)$.

b. Determine the rate of change between the points $(-1, -1)$ and $(1, -1)$.

c. **Construct viable arguments.** Is this a linear function? Justify your answer.

4. a. Determine the slopes of the lines shown.

b. **Express regularity in repeated reasoning.** Describe the slope of any line that rises as you view it from left to right.
Lesson 9-3
More About Slopes

5. a. Determine the slopes of the lines shown.

b. **Express regularity in repeated reasoning.** Describe the slope of any line that falls as you view it from left to right.

6. a. Determine the slopes of the lines below.

b. What is the slope of a horizontal line?

7. a. Determine the slopes of the lines shown.

b. What is the slope of a vertical line?
8. Summarize your findings in Items 4—7. Tell whether the slopes of the lines described in the table below are positive, negative, 0, or undefined.

<table>
<thead>
<tr>
<th>Up from left to right</th>
<th>Down from left to right</th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
</table>

9. Suppose you are given several points on the graph of a function. Without graphing, how could you determine whether the function is linear?

10. How can you tell from a graph if the slope of a line is positive or negative?

11. Describe a line having an undefined slope. Why is the slope undefined?

**LESSON 9-3 PRACTICE**

12. **Make use of structure.** Sketch a line for each description.
   a. The line has a positive slope.
   b. The line has a negative slope.
   c. The line has a slope of 0.

13. Does the table represent a linear function? Justify your answer.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>19</td>
</tr>
<tr>
<td>11</td>
<td>29</td>
</tr>
</tbody>
</table>

14. Are the points (12, 11), (2, 7), (5, 9), and (1, 5) part of the same linear function? Explain.
ACTIVITY 9 PRACTICE
Write your answers on notebook paper.
Show your work.

Lesson 9-1

1. Find $\triangle x$ and $\triangle y$ for each of the following pairs of points.
   a. $(2, 6), (-6, -8)$
   b. $(0, 9), (4, -8)$
   c. $(-3, -3), (7, 10)$

For Items 2 and 3, use the table to calculate the slope.

2. 
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -5 & -1 \\
   0 & 2 \\
   5 & 5 \\
   10 & 8 \\
   \end{array}
   \]

3. 
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -4 & 20 \\
   -3 & 14 \\
   0 & -4 \\
   2 & -16 \\
   \end{array}
   \]

4. Two points on a line are $(-10, 1)$ and $(5, -5)$. If the $y$-coordinate of another point on the line is $-3$, what is the $x$-coordinate?

For Items 5—7, determine the slope of the line that passes through each pair of points.

5. $(-4, 11)$ and $(1, -9)$
6. $(-10, -3)$ and $(-5, 1)$
7. $(-2, -7)$ and $(-8, -4)$
8. Are the three points $(2, 3), (5, 6),$ and $(0, -2)$ on the same line? Explain.

9. Which of the following pairs of points lies on a line with a slope of $-\frac{3}{5}$?
   A. $(4, 0), (-2, 10)$
   B. $(4, 2), (10, 4)$
   C. $(-4, -10), (0, -2)$
   D. $(10, -2), (0, 4)$

For Item 10, determine the slope of the line that is graphed.

10. 
   <Graph image>

Lesson 9-2

11. Juan earns $7 per hour plus $20 per week making picture frames.
   a. Write a function $g(x)$ for Juan’s total earnings if he works $x$ hours in one week.
   b. Without graphing the function, determine the slope.
   c. Describe the meaning of the slope within the context of Juan’s job.

12. The graph shows the height of an airplane as it descends to land.
   <Graph image>
   a. Does the function have a constant rate of change? If so, what is it?
   b. What is the slope of the line?
   c. How are the rate of change and the slope of the line related?
   d. Describe the meaning of the slope within the context of the situation.
Lesson 9-3

For Items 13–15, tell whether the function is linear. Justify your response.

13.  
\[ \begin{array}{c|c} 
 x & y \\
-3 & 44 \\
-1 & 4 \\
0 & -1 \\
1 & 4 \\
\end{array} \]

14.  
\[ \begin{array}{c|c} 
 x & y \\
-5 & -7 \\
0 & -8 \\
5 & -9 \\
10 & -10 \\
\end{array} \]

15.  
\[ \begin{array}{c|c} 
 x & y \\
4 & -30 \\
6 & -46 \\
8 & -62 \\
9 & -70 \\
\end{array} \]

16. One point on the line described by \( y = -2x + 3 \) is shown below. Use your knowledge of slope to give the coordinates of three more points on the line.

17. Which of the following is not a linear function?
   A. (4, -6), (7, -12), (8, -14), (10, -18), (2, -2)
   B. (-2, -6), (1, 0), (4, -30), (0, 2), (7, -96)
   C. (-4, 9), (0, 7), (2, 6), (6, 4), (8, 3)
   D. (2, 18), (6, 50), (-3, -22), (0, 2), (3, 26)

For Items 18 and 19, identify the slope of the line in each graph as positive, negative, 0, or undefined.

18.  
[Graph]

19.  
[Graph]

20. The slope of a line is 0. It passes through the point (-3, 4). Identify two other points on the line. Justify your answers.

MATHEMATICAL PRACTICES
Look For and Make Use of Structure

21. Describe three different ways to determine the slope of a line and the similarities and differences between the methods.
Learning Targets:
- Write and graph direct variation.
- Identify the constant of variation.

SUGGESTED LEARNING STRATEGIES: Create Representations, Interactive Word Wall, Marking the Text, Sharing and Responding, Discussion Groups

You work for a packaging and shipping company. As part of your job there, you are part of a package design team deciding how to stack boxes for packaging and shipping. Each box is 10 cm high.

1. Complete the table and make a graph of the data points (number of boxes, height of the stack).

<table>
<thead>
<tr>
<th>Number of Boxes</th>
<th>Height of the Stack (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

2. Write a function to represent the data in the table and graph above.

3. What is a reasonable and realistic domain for the function? Explain.

4. What is a reasonable and realistic range for the function? Explain.
5. What do \(f(x)\), or \(y\), and \(x\) represent in your equation from Item 2?

6. Describe any patterns that you notice in the table and graph representing your function.

7. The number of boxes is directly proportional to the height of the stack. Use a proportion to determine the height of a stack of 12 boxes.

When two values are directly proportional, there is a direct variation. In terms of stacking boxes, the height of the stack varies directly as the number of boxes.

8. Using variables \(x\) and \(y\) to represent the two values, you can say that \(y\) varies directly as \(x\). Use your answer to Item 6 to explain this statement.

9. Direct variation is defined as \(y = kx\), where \(k \neq 0\) and the coefficient \(k\) is the constant of variation.
   a. Consider your answer to Item 2. What is the constant of variation in your function?
   
   b. Why do you think the coefficient is called the constant of variation?
   
   c. Reason quantitatively. Explain why the value of \(k\) cannot be equal to 0.
   
   d. Write an equation for finding the constant of variation by solving the equation \(y = kx\) for \(k\).
10. a. Interpret the meaning of the point (0, 0) in your table and graph.

b. True or False? Explain your answer. “The graphs of all direct variations are lines that pass through the point (0, 0).”

c. Identify the slope and \( y \)-intercept in the graph of the stacking boxes.

d. Describe the relationship between the constant of variation and the slope.

Direct variation can be used to answer questions about stacking and shipping your boxes.

11. The height \( y \) of a different stack of boxes varies directly as the number of boxes \( x \). For this type of box, 25 boxes are 500 cm high.

   a. Find the value of \( k \). Explain how you found your answer.

   b. Write a direct variation equation that relates \( y \), the height of the stack, to \( x \), the number of boxes in the stack.

   c. How high is a stack of 20 boxes? Explain how you would use your direct variation equation to find the height of the stack.
12. At the packaging and shipping company, you get paid each week. One week you earned $48 for 8 hours of work. Another week you earned $30 for 5 hours of work.
   a. Write a direct variation equation that relates your wages to the number of hours you worked each week. Explain the meaning of each variable and identify the constant of variation.

   b. How much would you earn if you worked 3.5 hours in one week?

13. Tell whether the tables, graphs, and equations below represent direct variations. Justify your answers.

   a. $y = 20x$
   b. $y = 3x + 2$
   c. $\begin{array}{c|c}
       x & y \\
       \hline
       2 & 12 \\
       4 & 24 \\
       6 & 36 \\
   \end{array}$
   d. $\begin{array}{c|c}
       x & y \\
       \hline
       2 & 8 \\
       4 & 12 \\
       6 & 16 \\
   \end{array}$
Lesson 10-1
Direct Variation

LESSON 10-1 PRACTICE

14. In the equation \( y = 15x \), what is the constant of variation?

15. In the equation \( y = 8x \), what is the constant of variation?

16. The value of \( y \) varies directly with \( x \) and the constant of variation is 7. What is the value of \( x \) when \( y = 63 \)?

17. The value of \( y \) varies directly with \( x \) and the constant of variation is 12. What is the value of \( y \) when \( x = 5 \)?

18. Model with mathematics. The height of a stack of boxes varies directly with the number of boxes. A stack of 12 boxes is 15 feet high. How tall is a stack of 16 boxes?

19. Jan's pay is in direct variation to the hours she works. Jan earns $54 for 12 hours of work. How much will she earn for 18 hours work?
Learning Targets:

- Write and graph indirect variations.
- Distinguish between direct and indirect variation.

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Sharing and Responding, Think-Pair-Share, Discussion Groups

When packaging a different product, your team at the packaging and shipping company determines that all boxes for this product will have a volume of 400 cubic inches and a height of 10 inches. The lengths and the widths will vary.

1. To explore the relationship between length and width, complete the table and make a graph of the points.

<table>
<thead>
<tr>
<th>Width (x)</th>
<th>Length (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

2. How are the lengths and widths in Item 1 related? Write an equation that shows this relationship.

3. Use the equation you wrote in Item 2 to write a function to represent the data in the table and graph above.

4. Describe any patterns that you notice in the table and graph representing your function.
Lesson 10-2
Indirect Variation

In terms of box dimensions, the length of the box varies indirectly as the width of the box. Therefore, this function is called an **indirect variation**.

5. Recall that direct variation is defined as $y = kx$, where $k \neq 0$ and the coefficient $k$ is the constant of variation.
   a. How would you define indirect variation in terms of $y$, $k$, and $x$?

   b. Are there any limitations on these variables as there are on $k$ in direct variation? Explain.

   c. Write an equation for finding the constant of variation by solving for $k$ in your answer to Part (a).

6. **Reason abstractly.** Compare and contrast the equations of direct and indirect variation.

7. Compare and contrast the graphs of direct and indirect variation.

8. Use your function in Item 3 to determine the following measurements for your company.
   a. Find the length of a box whose width is 80 inches.

   b. Find the length of a box whose width is 0.4 inches.

MATH TIP

Indirect variation is also known as **inverse variation**.
9. The time, \( y \), needed to load the boxes on a truck for shipping varies indirectly as the number of people, \( x \), working. If 10 people work, the job is completed in 20 hours.
   \[ a. \] Explain how to find the constant of variation. Then find it.

   \[ b. \] Write an indirect variation equation that relates the time to load the boxes to the number of people working.

   \[ c. \] How long does it take 8 people to finish loading the boxes? Use your equation to answer this question.

   \[ d. \] On the grid below, make a graph to show the time needed for 2, 4, 5, 8, 10, and 25 people to load the boxes on the truck.

![Graph](image)

10. The cost for the company to ship the boxes varies indirectly with the number of boxes being shipped. If 25 boxes are shipped at once, it will cost $10 per box. If 50 boxes are shipped at once, the cost will be $5 per box.
   \[ a. \] Write an indirect variation equation that relates the cost per box to the number of boxes being shipped.

   \[ b. \] How much would it cost to ship only 10 boxes?

11. Is an indirect variation function a linear function? Explain.
Lesson 10-2
Indirect Variation

Check Your Understanding

12. Identify the following graphs as direct variation, indirect variation, neither, or both.

   a. [Graph 1]
   
   b. [Graph 2]
   
   c. [Graph 3]

13. Which equations are examples of indirect variation? Justify your answers.
   A. \( y = 2x \)
   B. \( y = \frac{x}{2} \)
   C. \( y = \frac{2}{x} \)
   D. \( xy = 2 \)

14. In the equation \( y = \frac{80}{x} \), what is the constant of variation?

LESSON 10-2 PRACTICE

15. Graph each function. Identify whether the function is an indirect variation.

   a. [Table for a]
   
   b. [Table for b]

16. Make sense of problems. For Parts (a) and (b) below, \( y \) varies indirectly as \( x \).
   a. If \( y = 6 \) when \( x = 24 \), find \( y \) when \( x = 16 \).
   b. If \( y = 8 \) when \( x = 20 \), find the value of \( k \).
Learning Targets:

- Write, graph, and analyze a linear model for a real-world situation.
- Interpret aspects of a model in terms of the real-world situation.

**SUGGESTED LEARNING STRATEGIES:** Marking the Text, Discussion Groups, Create Representations, Guess and Check, Use Manipulatives

Your design team at the packaging and shipping company has been asked to design a cardboard box to use when packaging paper cups for sale. Your supervisor has given you the following requirements.

- All lateral faces of the container must be rectangular.
- The base of the container must be a square, just large enough to accommodate one cup.
- The height of the container must be given as a function of the number of cups the container will hold.
- All measurements must be in centimeters.

To help discover which features of the cup affect the height of the stack, collect data on two types of cups found around the office.

1. **Use appropriate tools strategically.** Use two different types of cups to complete the tables below.

   **CUP 1**
   
<table>
<thead>
<tr>
<th>Number of Cups</th>
<th>Height of Stack</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

   **CUP 2**
   
<table>
<thead>
<tr>
<th>Number of Cups</th>
<th>Height of Stack</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

2. **Express regularity in repeated reasoning.** What patterns do you notice that might help you figure out the relationship between the height of the stack and the number of cups in that stack?
Use your data for Cup 1 to complete Items 3–13.

3. Make a graph of the data you collected.

4. Predict, without measuring, the height of a stack of 16 cups. Explain how you arrived at your prediction.

5. Predict, without measuring, the height of a stack of 50 cups. Explain how you arrived at your prediction.

6. Write an equation that gives the height of a stack of cups, \( h \), in terms of \( n \), the number of cups in the stack.

7. Use your equation from Item 6 to find \( h \) when \( n = 16 \) and when \( n = 50 \). Do your answers to this question agree with your predictions in Items 4 and 5?
8. Sketch the graph of your equation from Item 6.

9. How are the graphs you made in Items 3 and 8 the same? How are they different?

10. Do the graphs in Items 3 and 8 represent direct variation, indirect variation, or neither? Explain.

11. Remember that you are designing a container with a square base. What dimension(s), other than the height of the stack, do you need to design your cup container? Use Cup 1 to find this/these dimension(s).

12. Find the dimensions of a container that will hold a stack of 25 cups.
Lesson 10-3
Another Linear Model

13. Your team has been asked to communicate its findings to your supervisor. Write a report to her that summarizes your findings about the cup container design. Include the following information in your report.
   • The equation your team discovered to find the height of the stack of Cup 1 style cups
   • A description of how your team discovered the equation and the minimum number of cups needed to find it
   • An explanation of how the numbers in the equation relate to the physical features of the cup
   • An equation that could be used to find the height of the stack of Cup 2 style cups

Check Your Understanding

14. A group of students performed the cup activity described in this lesson. For their Cup 1, they found the equation \( h = 0.25n + 8.5 \), where \( h \) is the height in inches of a stack of cups and \( n \) is the number of cups.
   a. What would be the height of 25 cups? Of 50 cups?
   b. Graph this equation. Describe your graph.

LESSON 10-3 PRACTICE

15. Reason quantitatively. A group of students performed the cup activity in this lesson using plastic drinking cups. Their data is shown below.

<table>
<thead>
<tr>
<th>Number of Cups</th>
<th>Height of Stack</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.5 cm</td>
</tr>
<tr>
<td>2</td>
<td>16 cm</td>
</tr>
<tr>
<td>3</td>
<td>17.5 cm</td>
</tr>
<tr>
<td>4</td>
<td>19 cm</td>
</tr>
<tr>
<td>5</td>
<td>20.5 cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Cups</th>
<th>Height of Stack</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.5 cm</td>
</tr>
<tr>
<td>2</td>
<td>11.75 cm</td>
</tr>
<tr>
<td>3</td>
<td>13 cm</td>
</tr>
<tr>
<td>4</td>
<td>14.25 cm</td>
</tr>
<tr>
<td>5</td>
<td>15.5 cm</td>
</tr>
</tbody>
</table>

For each cup, write and graph an equation. Describe your graphs.

16. A consultant earns a flat fee of $75 plus $50 per hour for a contracted job. The table shows the consultant’s earnings for the first four hours she works.

<table>
<thead>
<tr>
<th>Hours</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$75</td>
</tr>
<tr>
<td>1</td>
<td>$125</td>
</tr>
<tr>
<td>2</td>
<td>$175</td>
</tr>
<tr>
<td>3</td>
<td>$225</td>
</tr>
<tr>
<td>4</td>
<td>$275</td>
</tr>
</tbody>
</table>

The consultant has a 36-hour contract. How much will she earn?
Learning Targets:
• Write the inverse function for a linear function.
• Determine the domain and range of an inverse function.

**SUGGESTED LEARNING STRATEGIES:** Visualization, Create Representations, Think-Pair-Share, Discussion Groups, Construct an Argument

After reading your report, your supervisor was able to determine the equation for the height of the stack for the specific cup that the company will manufacture. The company will use the function \( S(n) = 0.5n + 12.5 \).

1. What do \( S \), \( n \), and \( S(n) \) represent?

2. What do the numbers 0.5 and the 12.5 in the function \( S \) tell you about the physical features of the cup?

3. Evaluate \( S(1) \) to find the height of a single cup.

4. How tall is a stack of 35 cups? Show your work using function notation.

5. If you add 2 cups to a stack, by how much does the height of the stack increase?

6. If you add 20 cups to a stack, by how much does the height of the stack increase?

7. **Critique the reasoning of others.** A member of one of the teams stated: “If you double the number of cups in a stack, then the height of the stack is also doubled.” Is this statement correct? Explain.
8. If you were to graph the function $S(n) = 0.5n + 12.5$, you would see that the points lie on a line.
   a. What is the slope of this line?
   b. Interpret the slope of the line as a rate of change that relates a change in height to a change in the number of cups.

9. Write an equation for $y$ in terms of $x$.

10. Explain how the numbers in your equation relate to the numbers in the table.

11. Evaluate the function you wrote in Item 9 for each of the following values of $x$.
   a. $x = 8$  
   b. $x = 12$  
   c. $x = 15$  
   d. $x = 0$

12. a. The supervisor wanted to increase the height of a container by 5 cm. How many more cups would fit in the container?

   b. If the supervisor wanted to increase the height of a container by 6.4 cm, how many more cups would fit in the container?

   c. How many cups fit in a container that is 36 cm tall?

   d. How many cups fit in a container that is 50 cm tall?
13. The function \( S(n) = 0.5n + 12.5 \) describes the height \( S \) in terms of the number of cups \( n \).

   a. Solve this equation for \( n \) to describe the number of cups \( n \) in terms of the height \( S \).

   b. How many cups fit in a carton that is 85 cm tall? Compare your method of answering this question to your method used in Items 12c and 12d.

   c. What is the slope of the line represented by your equation in Part (a)? Interpret it as a rate of change and compare it to the rate of change found in Item 8b.

An inverse function is a function that interchanges the independent and dependent variables of another function. In Item 13, you found the inverse function for \( S(n) \). In general, the inverse function for \( f(x) \) is \( f^{-1}(x) \).

**Example A**

Use the table below to fill in the steps to find the inverse function for \( f(x) = 2x + 3 \).

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write the function, replacing ( f(x) ) with ( y ).</td>
<td></td>
</tr>
<tr>
<td>Switch ( x ) and ( y ).</td>
<td></td>
</tr>
<tr>
<td>Solve for ( y ) in terms of ( x ).</td>
<td></td>
</tr>
<tr>
<td>Replace ( y ) with ( f^{-1}(x) ).</td>
<td></td>
</tr>
</tbody>
</table>

**Try These A**

Determine the inverses of each of the following of functions.

a. \( f(x) = -4x - 5 \)  
   b. \( f(x) = \frac{2}{3}x + 2 \)  
   c. \( f(x) = -\frac{1}{2}x + 4 \)
Lesson 10-4
Inverse Functions

Only those functions that are one-to-one functions have an inverse function. Functions that are not one-to-one must have their domain restricted for an inverse function to exist.

14. Is \( S(n) = 0.5n + 12.5 \) a one-to-one function? Explain.

15. Do the following graphs of functions show one-to-one functions? Justify your answers.

a. 

\[ \begin{array}{c|c|c|c}
 x & 1 & 2 & 3 \\
 y & 5 & 4 & 3 \\
\end{array} \]

b. 

\[ \begin{array}{c|c|c|c}
 x & 1 & 2 & 3 \\
 y & 10 & 8 & 6 \\
\end{array} \]

A visual test for a one-to-one function is the horizontal line test. If you can draw a horizontal line that intersects the graph of a function in more than one place, that function is not one-to-one.

17. A function is defined by the ordered pairs \{(-3, -1), (-1, 0), (1, 1), (3, 2), (5, 3)\}. What are the domain and range of the function?

Because inputs and outputs are switched when writing the inverse of a function, the domain of a function is the range of its inverse function, and the range of a function is the domain of its inverse function.

18. What are the domain and range of the inverse function for the function in Item 17?

The function \(f(x) = 2.5x + 3.5\) gives the cost \(f(x)\) of a cab ride of \(x\) miles.

19. What is the cost of a 6-mile ride?

20. What are the reasonable domain and range of the function?

21. Write the inverse function, \(f^{-1}(x)\). What are the domain and range of \(f^{-1}(x)\)?

22. What does \(x\) represent in the inverse function?

23. A cab ride costs $46. Show how to use the inverse function to find the distance of the cab ride in miles.

**Check Your Understanding**

The function \(f(x) = 2.5x + 3.5\) gives the cost \(f(x)\) of a cab ride of \(x\) miles.

19. What is the cost of a 6-mile ride?

20. What are the reasonable domain and range of the function?

21. Write the inverse function, \(f^{-1}(x)\). What are the domain and range of \(f^{-1}(x)\)?

22. What does \(x\) represent in the inverse function?

23. A cab ride costs $46. Show how to use the inverse function to find the distance of the cab ride in miles.

**LESSON 10-4 PRACTICE**

Make use of structure. Find the inverse function, \(f^{-1}(x)\), for the functions in Items 24–26.

24. \(f(x) = 3x - 5\)

25. \(f(x) = -2x + 10\)

26. \(f(x) = \frac{7x}{3} - \frac{1}{6}\)

27. The yearly membership fee for the Art Museum is $75. After paying the membership fee, the cost to enter each exhibit is $7.50.
   a. Write a function for the total cost of a member for one year of attending the art museum.
   b. What is the total cost for a member who sees 12 exhibits?
   c. What are the domain and range for the function?
   d. What is \(f^{-1}(x)\)? What are the domain and range for \(f^{-1}(x)\)?
   e. What does \(x\) represent in \(f^{-1}(x)\)?
   f. How many exhibits can a member see in a year for a total of $210, including the membership fee?
**ACTIVITY 10 PRACTICE**

Write your answers on notebook paper.
Show your work.

**Lesson 10-1**

1. The value of \( y \) varies directly as \( x \) and \( y = 125 \) when \( x = 25 \). What is the value of \( y \) when \( x = 2? \)

2. Which is the graph of a direct variation?

   - A.  
   
   ![Graph A]

   - B.  
   
   ![Graph B]

   - C.  
   
   ![Graph C]

   - D.  
   
   ![Graph D]

3. Which equation does not represent a direct variation?
   - A. \( y = \frac{x}{3} \)
   - B. \( y = \frac{2}{5}x \)
   - C. \( y = \frac{3}{x} \)
   - D. \( y = \frac{5x}{2} \)

4. The value of \( y \) varies directly as \( x \) and \( y = 9 \) when \( x = 6 \). What is the value of \( y \) when \( x = 15? \)

5. The tailor determines that the cost of material varies directly with the amount of material. The cost is $42 for 14 yards of material. What is the cost for 70 yards of material?

**Lesson 10-2**

6. The value of \( y \) varies indirectly as \( x \) and \( y = 4 \) when \( x = 20 \). What is the value of \( y \) when \( x = 40? \)
   - A. \( y = 2 \)
   - B. \( y = 8 \)
   - C. \( y = 50 \)
   - D. \( y = 80 \)

7. The temperature varies indirectly as the distance from the city. The temperature equals 3°C when the distance from the city is 40 miles. What is the temperature when the distance is 20 miles from the city?

8. The amount of gas left in the gas tank of a car varies indirectly to the number of miles driven. There are 9 gallons of gas left after 24 miles. How much gas is left after the car is driven 120 miles?
Lesson 10-3

The Pete’s Pets chain of pet stores is growing. The table below shows the number of stores in business each month. Use the table for Items 9–12.

<table>
<thead>
<tr>
<th>Month</th>
<th>Stores</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>0</td>
</tr>
<tr>
<td>February</td>
<td>3</td>
</tr>
<tr>
<td>March</td>
<td>6</td>
</tr>
<tr>
<td>April</td>
<td>9</td>
</tr>
<tr>
<td>May</td>
<td>12</td>
</tr>
<tr>
<td>June</td>
<td>15</td>
</tr>
</tbody>
</table>

9. According to the table, how many new stores open per month?
10. How many stores will be in business by December?
11. Are the Pete’s Pets data an example of indirect variation, direct variation, or neither? Explain your reasoning.
12. What is the slope of this function? Interpret the meaning of the slope.

Jeremy collected the following data on stacking chairs. Use the data for Items 13 and 14.

<table>
<thead>
<tr>
<th>Number of Chairs</th>
<th>Height (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>39</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
</tr>
<tr>
<td>6</td>
<td>45</td>
</tr>
</tbody>
</table>

13. Write a linear function that models the data.
14. Chairs cannot be stacked higher than 5 feet. What is the maximum number of chairs Jeremy can stack? Justify your answer.

Lesson 10-4

Write the inverse function for each of the following.
15. \( f(x) = -8x + 4 \)
16. \( f(x) = \frac{1}{4}x - 3 \)
17. \( f(x) = 8x - 15 \)
18. \( f(x) = x + 1 \)
19. \( f(x) = -x + 1 \)

The formula to convert degrees Celsius \( C \) to degrees Fahrenheit \( F \) is \( F = \frac{9}{5}C + 32 \). Use this formula for Items 20–23.
20. Use the formula to convert 100°C to degrees Fahrenheit.
21. What is the slope?
22. The temperature is 50°F. What is the temperature in degrees Celsius?
23. Solve for \( C \) to derive the formula that converts degrees Fahrenheit to degrees Celsius.

MATHEMATICAL PRACTICES

Look for and Make Use of Structure

24. Describe the similarities and differences between finding the inverse of a function and working backward to solve a problem.
Learning Targets:
- Identify sequences that are arithmetic sequences.
- Use the common difference to determine a specified term of an arithmetic sequence.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Discussion Groups, Marking the Text, Use Manipulatives

1. Use toothpicks to make the following models.

   ![Stage 1](image1)
   ![Stage 2](image2)
   ![Stage 3](image3)

2. Continue to Stage 4 and Stage 5. Draw your models below.

   ![Stage 4](image4)
   ![Stage 5](image5)

3. Complete the table for the number of toothpicks used for each stage of the models up through Stage 5.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Toothpicks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
</tr>
</tbody>
</table>
Lesson 11-1
Identifying Arithmetic Sequences

4. Model with mathematics. Use the following grid to make a graph of the data in the table.

```
<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Toothpicks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
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<tr>
<td>4</td>
<td>40</td>
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<td>7</td>
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<tr>
<td>8</td>
<td>80</td>
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<tr>
<td>9</td>
<td>90</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>
```

a. Is your graph discrete or continuous? Explain your answer.

b. Is your graph the graph of a linear function? Explain your answer.

An ordered list of numbers is called a sequence. The numbers in a sequence are terms. To refer to the \( n \)th term in a sequence, you can use either function notation, \( f(n) \), or the indexed variable \( a_n \).

The toothpick data form a sequence. The numbers of toothpicks at each stage are the terms of the sequence.

5. What are the first four terms of the toothpick sequence?
   \( a_1 = \quad a_2 = \quad a_3 = \quad a_4 = \)

6. In a sequence, what is the distinction between a term and a term number?

Check Your Understanding

7. For the sequence 7, −5, −3, 1, 1, . . . , what is \( a_4 \)?

8. For the sequence 1, 5, 9, 13, 17, . . . , what is \( a_5 \)?

An arithmetic sequence is a sequence in which the difference between terms is constant. The difference between consecutive terms in an arithmetic sequence is called the common difference.

9. Explain why the toothpick sequence is an arithmetic sequence.
Lesson 11-1
Identifying Arithmetic Sequences

10. What is the rate of change for the toothpick data?

11. Look back at the graph in Item 4.
   a. Determine the slope between any two points on the graph.
   b. Describe the connections between the slope, the rate of change, and
      the common difference.

Check Your Understanding

Tell whether each sequence is an arithmetic sequence. For each arithmetic
sequence, find the common difference.

12. 9, 16, 23, 30, 37, . . .
13. −24, −20, −14, −10, −4, 0, . . .
14. −2.8, −2.2, −1.6, −1.0, . . .
15. 3, 5, 8, 12, 17, . . .
16. $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, . . .
17. **Reason abstractly.** Can the common difference in an arithmetic
    sequence be negative? If so, give an example. If not, explain why not.

**LESSON 11-1 PRACTICE**

Tell whether each sequence is arithmetic. If the sequence is arithmetic,
identify the common difference and find the indicated term.

18. −9, −4, 1, 6, 11, . . . ; $a_7 = ?$
19. 2, 4, 7, 11, 16, 22, . . . ; $a_9 = ?$
20. −7, −1, 5, 11, 17, . . . ; $a_6 = ?$
21. 1.2, 1.9, 2.6, 3.3, 4.0, . . . ; $a_8 = ?$
22. 3, $\frac{5}{2}$, $\frac{3}{2}$, $-\frac{3}{2}$, . . . ; $a_7 = ?$
23. Write an arithmetic sequence in which the last digit of each term is 4.
    What is the common difference for your sequence?
24. **Critique the reasoning of others.** Jim said that the terms in an
    arithmetic sequence must always increase, because you must add the
    common difference to each term to get the next term. Is Jim correct?
    Justify your reasoning.
Learning Targets:

- Develop an explicit formula for the $n$th term of an arithmetic sequence.
- Use an explicit formula to find any term of an arithmetic sequence.
- Write a formula for an arithmetic sequence given two terms or a graph.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Interactive Word Wall, Predict and Confirm, Think-Pair-Share

1. Rewrite the terms of the toothpick sequence and identify the common difference.

2. Find the next three terms in the sequence without building toothpick models. Explain how you found your answers.

3. Why might it be difficult to find the 100th term of the toothpick sequence using repeated addition of the common difference?

An explicit formula for a sequence allows you to compute any term in a sequence without computing all of the terms before it.

4. Develop an explicit formula for the toothpick sequence using the first term and the common difference.

The first term of the sequence is $a_1 = 8$.

The second term is $a_2 = 8 + 6$.

The third term is $a_3 = 8 + 6 + 6$, or $a_3 = 8 + 2(6)$.

a. Write an expression for the fourth term using the value of $a_1$ and the common difference.

$b =

b. Express regularity in repeated reasoning. Use the patterns you have observed to determine the 15th term. Justify your reasoning.

$c =

$
Lesson 11-2
A Formula for Arithmetic Sequences

The formula you wrote in Item 4c is the explicit formula for the toothpick sequence.

For any arithmetic sequence, \(a_1\) refers to the first term and \(d\) refers to the common difference.

**5.** Write an explicit formula for finding the \(n\)th term of any arithmetic sequence.

**Example A**
Write the explicit formula for the arithmetic sequence 3, −3, −9, −15, −21, . . . . Then use the formula to find the value of \(a_{10}\).

**Step 1:** Find the common difference.

\[
\begin{align*}
3 & \quad -3 \\
-6 & \quad -6 \\
-9 & \quad -6 \\
-15 & \quad -6 \\
-21 & \quad -6
\end{align*}
\]

The common difference is −6.

**Step 2:** Write the explicit formula and simplify.

\[
\begin{align*}
\quad & a_n = a_1 + (n - 1)d \\
\quad & a_n = 3 + (n - 1)(-6) = 9 - 6n
\end{align*}
\]

**Step 3:** Use the formula to find \(a_{10}\) by substituting for \(n\).

\[
\begin{align*}
a_{10} = 9 - 6(10) = -51
\end{align*}
\]

**Solution:** The explicit formula is \(a_n = 9 - 6n\) and \(a_{10} = -51\).

**Try These A**
For the following arithmetic sequences, find the explicit formula and the value of the indicated term.

**a.** 2, 6, 10, 14, 18, . . . ; \(a_{21}\)

**b.** −0.6, −1.0, −1.4, −1.8, −2.2, . . . ; \(a_{15}\)

**c.** \(\frac{1}{3}\), 1, \(\frac{5}{3}\), \(\frac{7}{3}\), . . . ; \(a_{37}\)

An arithmetic sequence can be graphed on a coordinate plane. In the ordered pairs the term numbers (1, 2, 3, . . . ) are the \(x\)-values and the terms of the sequence are the \(y\)-values.

**6.** Look back at the sequence in Example A. Make a prediction about its graph.
7. On the grid below, create a graph of the arithmetic sequence in Example A. Revise your prediction in Item 6 if necessary.

8. Determine the slope between any two points on your graph in Item 7. How does the slope compare to the common difference of the sequence?

9. The first three terms of the arithmetic sequence 2, 5, 8, \ldots are graphed below. Determine the common difference. Then graph the next three terms.

10. Determine the slope between any two points you graphed in Item 9. How does the slope compare to the common difference of the sequence?

11. Write the explicit formula for the sequence graphed in Item 9.

12. If you are given a graph of an arithmetic sequence, how do you find the explicit formula?

The 11th term of an arithmetic sequence is 59 and the 14th term is 74.

13. **Reason quantitatively.** How could you determine the value of \( d \)? What is the value of \( d \)?
Lesson 11-2
A Formula for Arithmetic Sequences

14. How could you determine the value of $a_1$? What is this value?

15. Write the explicit formula for the $n$th term of the sequence.

16. Determine $a_{30}$, the 30th term of the sequence.

Check Your Understanding

17. The explicit formula for the $n$th term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$. What does each variable in the explicit formula represent?

18. What is $a_{30}$ of the arithmetic sequence 45, 40, 35, 30, . . .?

19. The 8th term of an arithmetic sequence is 12.5 and the 13th term is 20. What is $a_{25}$?

20. Construct viable arguments. Could an arithmetic sequence also be a direct variation? Justify your answer.

21. Why is the graph of an arithmetic sequence made up of discrete points?

**LESSON 11-2 PRACTICE**

For the following arithmetic sequences, find the explicit formula and the value of the term indicated.

22. 2, 11, 20, 29, . . .; $a_{30}$

23. 0.5, 0.75, 1, . . .; $a_{18}$

24. $\frac{1}{6}$, 0, $-\frac{1}{6}$, $-\frac{1}{3}$, . . .; $a_{42}$

25. The 3rd term of an arithmetic sequence is $-1$ and the 7th term is $-13$. Find the explicit formula for this sequence. What is $a_{22}$?

26. Make use of structure. Write the explicit formula for each arithmetic sequence graphed below. Then find the 25th term.

a. ![Graph a](image)

b. ![Graph b](image)

c. ![Graph c](image)
Learning Targets:
- Use function notation to write a general formula for the \( n \)th term of an arithmetic sequence.
- Find any term of an arithmetic sequence written as a function.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Discussion Groups, Sharing and Responding, Group Presentation

An arithmetic sequence is a special case of a linear function. The terms of the sequence are the functional values \( f(1), f(2), f(3), \ldots, f(n) \) for some \( n \).

1. Fill in the next three terms of the arithmetic sequence.
   
   \[
   \begin{align*}
   a_1 &= 7 = f(1) \\
   a_2 &= 10 = f(2) \\
   a_3 &= 13 = f(3) \\
   a_4 &= \underline{16} = f(4) \\
   a_5 &= \underline{19} = f(5) \\
   a_6 &= \underline{22} = f(6)
   \end{align*}
   \]

2. What is the \( n \)th term of the sequence?

3. What function \( f \) could be used to describe the sequence?

4. What is the common difference of the sequence? How is the common difference related to the function you wrote in Item 3?

5. **Attend to precision.** Describe the domain of \( f \) using set notation. (*Hint: What values are used as inputs for \( f \)?*)

6. What ordered pair represents the \( n \)th term of the sequence?

7. Describe the graph of \( f \). How is the common difference related to the graph?
Lesson 11-3
Arithmetic Sequences as Functions

Check Your Understanding

For Items 8–11, use the arithmetic sequence –5, 1, 7, 13, . . . .

8. What is \( f(1) \)?
9. What is \( f(4) \)?
10. Write a function to describe the sequence.
11. Use your function to find \( f(14) \).

LESSON 11-3 PRACTICE

Write a function to describe each arithmetic sequence.

12. 10, 14, 18, 22, …
13. 8.5, 10.3, 12.1, 13.9, …
14. \( \frac{1}{3}, \frac{7}{12}, \frac{5}{6}, \frac{13}{12} \), …
15. –7, –4.5, –2, 0.5, …
16. \( \begin{align*}
   \text{y} & \quad \text{x} \\
   8 & \quad 1 \\
   7 & \quad 2 \\
   6 & \quad 3 \\
   5 & \quad 4 \\
   4 & \quad 5 \\
   3 & \quad 1 \\
   2 & \quad 2 \\
   1 & \quad 3 \\
   0 & \quad 4 \\
   -1 & \quad 5 \\
   -2 & \quad -1 \\
   -3 & \quad -2 \\
   -4 & \quad -3 \\
   -5 & \quad -4 \\
   -6 & \quad -5
\end{align*} \)

The 1st term of an arithmetic sequence is 5, and the common difference is 1.5.
Use this information for Items 17–19.

17. What is \( f(3) \)?
18. Write a function to describe this arithmetic sequence.
19. Determine the 25th term of the sequence. Use function notation in your answer.

20. Make sense of problems. The 3rd term of an arithmetic sequence is –2, and the 8th term is –32. Write a function to describe this sequence.
MATH TIP
In a sequence, \(f(n - 1) = a_{n-1}\) refers to the term before \(f(n) = a_n\). Item 1 is asking "For any value of \(n\), how can you find the term before \(f(n) = a_n\)?"

**Learning Targets:**
- Write a recursive formula for a given arithmetic sequence.
- Use a recursive formula to find the terms of an arithmetic sequence.

**SUGGESTED LEARNING STRATEGIES:** Look for a Pattern, Create Representations, Close Reading, Marking the Text, Discussion Groups

In a sequence, the term before \(f(n) = a_n\) is \(f(n - 1) = a_{n-1}\).

The first four terms of the toothpick sequence can be written as

\[
\begin{align*}
a_1 &= 8 \\
a_2 &= 14 = a_1 + 6 \\
a_3 &= 20 = a_2 + 6 \\
a_4 &= 26 = a_3 + 6
\end{align*}
\]

1. For any value of \(n\), how can you find the value of \(f(n - 1) = a_{n-1}\)?

A recursive formula can be used to represent an arithmetic sequence. Recursion is the process of choosing a starting term and repeatedly applying the same process to each term to arrive at the following term.

A recursive formula for an arithmetic sequence looks like this:

\[
\begin{align*}
a_1 &= \text{1st term} \\
a_n &= a_{n-1} + d \quad \text{or in function notation:} \quad f(1) &= \text{1st term} \\
f(n) &= f(n-1) + d
\end{align*}
\]

2. The recursive formulas for the toothpick sequence are partially given below. Complete them by writing the expressions for \(a_n\) and \(f(n)\).

\[
\begin{align*}
a_1 &= 8 \\
a_n &= \quad \text{and} \quad f(1) &= 8 \\
f(n) &= \quad \text{or} \quad f(n) = f(n-1) + 6
\end{align*}
\]

**Check Your Understanding**

Write the recursive formula for the following arithmetic sequences. Include the recursive formula in function notation.

3. 2, 4, 6, 8, . . .
4. -2, -5, -8, -11, . . .
5. -3, -\(\frac{3}{2}\), 0, \(\frac{3}{2}\), . . .
6. Suppose that \(a_{n-1} = -4\).
   a. Find the value of \(a_n\) for the arithmetic sequence with the recursive formula \[
\begin{align*}
a_1 &= 6 \\
a_n &= a_{n-1} + (-5) \\
f(n) &=
\end{align*}
\]
   b. What term did you find? (In other words, what is \(n\) equal to?)
7. An arithmetic sequence has the recursive formula below.
\[
\begin{align*}
  f(1) &= \frac{1}{2} \\
  f(n) &= f(n-1) + 2
\end{align*}
\]
   a. Determine the first five terms of the sequence.

   b. Write the explicit formula for the sequence using function notation.

8. An arithmetic sequence has the explicit formula \( a_n = 3n - 8 \).
   a. What are the values of \( a_1 \) and \( a_2 \)?

   b. How can you use the values of \( a_1 \) and \( a_2 \) to find \( d \)? What is \( d \)?

   c. Use your answers to Parts (a) and (b) to write the recursive formula for the sequence.

In the 12th century, Leonardo of Pisa, also known as Fibonacci, first described a sequence known as the **Fibonacci sequence**. The sequence can be described by the recursive formula below.
\[
\begin{align*}
  a_1 &= 1 \\
  a_2 &= 1 \\
  a_n &= a_{n-1} + a_{n-2}, \text{ for } n > 2
\end{align*}
\]
Notice that the first two terms of the sequence are 1 and that the expression describing \( a_n \) applies to those terms after the 2nd term.

9. Use the recursive formula to determine the first 10 terms of the Fibonacci sequence.

10. Is the Fibonacci sequence an arithmetic sequence? Justify your response.
11. **Attend to precision.** Compare and contrast the explicit and recursive formulas for an arithmetic sequence.

12. Explain how to find any term of the Fibonacci sequence.

**LESSON 11-4 PRACTICE**

Write the recursive formula for each arithmetic sequence. Include the recursive formula in function notation.

13. 1, 6, 11, 16, . . .

14. 1, 4, 7, 10, 13, . . .

15. \(a_n = 11 - 3n\)

16. \(a_n = \frac{1}{4} + \frac{3}{20}n\)

17. Given \(f(n - 1) = 1.2\), use the recursive formula below to find \(f(n)\).

\[
\begin{align*}
f(1) &= -0.6 \\
f(n) &= f(n-1) + 0.3
\end{align*}
\]

18. **Reason quantitatively.** Describe how each sequence is similar to the Fibonacci sequence. Then find the next two terms.

   a. 4, 4, 8, 12, 20, 32, . . .

   b. 2, 2, 4, 6, 10, 16, . . .
ACTIVITY 11 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 11-1
For Items 1–3, refer to the toothpick pattern shown below.

1. Copy and complete the table below.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Toothpicks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

2. Write the number of toothpicks as a sequence.

3. Is the sequence you wrote in Item 2 an arithmetic sequence? If so, determine the common difference. If not, explain why not.

4. Which of the following is not an arithmetic sequence?
   A. $\frac{1}{2}, 1, \frac{3}{2}, 2 \ldots$
   B. 11, 14, 17, 20, ...
   C. 2, 4, 8, 16, ...
   D. 5, 2, −1, −4, ...

For Items 5 and 6, find the common difference for each arithmetic sequence.

5. −2.3, −1.1, 0.1, 1.3, ...

6. −9, −13, −18, −23, ...

7. What are the next three terms in the arithmetic sequence −6, −10, −14, ...?

8. Write an arithmetic sequence in which some of the terms are whole numbers and the common difference is $\frac{3}{4}$.

Lesson 11-2
For Items 9 and 10, determine the explicit formula for each arithmetic sequence.

9. 3, −3, −9, −15, ...

10. 1, 6, 11, 16, ...

11. What is the 20th term of the arithmetic sequence: 7, 4, 1, ...?

12. The 9th and 10th terms of an arithmetic sequence are −24 and −30, respectively. What is the 30th term?

13. The 15th and 21st terms of an arithmetic sequence are −67 and −97, respectively. What is the 30th term?

14. The 9th and 14th terms of an arithmetic sequence are 23 and 33, respectively. What is the 1st term?

15. For the sequence graphed below, write the explicit formula. Then find the 57th term of the sequence.

Lesson 11-3
16. Write a function to describe the arithmetic sequence graphed below. Then find the 5th term of the sequence. Use function notation in your answer.
17. For an arithmetic sequence, \( f(1) = \frac{4}{5} \) and the common difference is \(-1\). What is \( f(20)\)?

A. \( 18 \frac{1}{5} \)

B. \( -\frac{1}{5} \)

C. \( -20 \frac{1}{5} \)

D. \( -18 \frac{1}{5} \)

18. An arithmetic sequence is described by the function \( f(n) = -3n + 7 \). Determine the first five terms of this sequence.

Lesson 11-4

19. What are the first five terms in the arithmetic sequence with the recursive formula below?

\[
\begin{align*}
a_1 &= 5 \\
a_n &= a_{n-1} + 4
\end{align*}
\]

20. What is the recursive formula for the arithmetic sequence described by the function below?

\( f(n) = \frac{1}{2}n - 2 \)

21. What is the explicit formula for the arithmetic sequence that has the recursive formula below?

\[
\begin{align*}
f(1) &= -2.5 \\
f(n) &= f(n-1) - 4
\end{align*}
\]

For Items 22 and 23, write the recursive formula for the arithmetic sequence that is graphed. Include the recursive formula in function notation.

22. [Graph of an arithmetic sequence]

23. [Graph of a linear function]

MATHEMATICAL PRACTICES

Make Sense of Problems and Persevere in Solving Them

24. The Lucas sequence is related to the Fibonacci sequence. The first five terms of the Lucas sequence are given below.

\[
\begin{align*}
a_1 &= 1 \\
a_2 &= 1 + 2 = 3 \\
a_3 &= 1 + 3 = 4 \\
a_4 &= 2 + 5 = 7 \\
a_5 &= 3 + 8 = 11
\end{align*}
\]

a. Beginning with \( a_2 \), what do you observe about the first addends in each sum? What do you observe about the second addends?

b. What are the next two terms of the Lucas sequence? Explain how you determined your answer.
Pedro is planning to add a text messaging feature to his cell phone plan. He has gathered information about the two different plans offered by his wireless phone company.

Plan A: $4.00 per month plus 4 cents for each message  
Plan B: 5 cents per message

1. Use the mathematics you have been studying in this unit to provide Pedro with the following information for each plan.
   a. Plan A
      - a table of data
      - a graph of the data
      - the linear function that fits this plan
      - the domain and range of the function
   b. Plan B
      - a table of data
      - a graph of the data
      - the linear function that fits this plan
      - the domain and range of the function

2. If Pedro sends 360 messages on average each month, which plan would you recommend that he choose? Support your recommendation using mathematical evidence.

3. If Pedro knows that his average usage is going to increase to 500 text messages per month, should he change to a different plan? Explain and justify your reasoning.

4. Explain whether either of the plans represents a direct variation.

5. Pedro’s friend Chenetta is considering another text messaging plan that advertises the following: “A one-time joining fee of $3.00 and $0.08 per message.”
   a. Write an explicit formula for the text messaging plan.
   b. Chenetta knows that she sends and receives about 1800 text messages per month. Use an example and other mathematical evidence to let Chenetta know if you think this plan would be a good deal for her.
### Scoring Guide

The solution demonstrates the following characteristics:

<table>
<thead>
<tr>
<th>Mathematics Knowledge and Thinking (Items 1, 4, 5a)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>✷ Clear and accurate understanding of linear models, including direct variation</td>
<td>✷ Largely correct understanding of linear models, including direct variation</td>
<td>✷ Partial understanding of linear models, including direct variation</td>
<td>✷ Inaccurate or incomplete understanding of linear models, including direct variation</td>
<td></td>
</tr>
<tr>
<td>✷ Effective understanding of arithmetic sequences</td>
<td>✷ Adequate understanding of arithmetic sequences</td>
<td>✷ Some difficulty with arithmetic sequences</td>
<td>✷ Little or no understanding of arithmetic sequences</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving (Items 2, 3, 5b)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>✷ Appropriate and efficient strategy that results in a correct answer</td>
<td>✷ Strategy that may include unnecessary steps but results in a correct answer</td>
<td>✷ Strategy that results in some incorrect answers</td>
<td>✷ No clear strategy when solving problems</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mathematical Modeling / Representations (Items 1, 5a)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>✷ Clear and accurate tables of real-world data, graphs of the data, and linear functions to model the data, including reasonable domain and range</td>
<td>✷ Correct tables of real-world data, graphs of the data, and linear functions to model the data, including reasonable domain and range</td>
<td>✷ Partially correct tables of real-world data, graphs of the data, and linear functions to model the data, including reasonable domain and range</td>
<td>✷ Inaccurate or incomplete tables of real-world data, graphs of the data, and linear functions to model the data, including reasonable domain and range</td>
<td></td>
</tr>
<tr>
<td>✷ Fluency in writing an explicit formula to model a real-world scenario</td>
<td>✷ Little difficulty writing an explicit formula to model a real-world scenario</td>
<td>✷ Some difficulty writing an explicit formula to model a real-world scenario</td>
<td>✷ Significant difficulty writing an explicit formula to model a real-world scenario</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reasoning and Communication (Items 2–4, 5b)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>✷ Precise use of appropriate math terms and language to make and justify a recommendation</td>
<td>✷ Appropriate recommendations with adequate justifications</td>
<td>✷ Misleading or confusing recommendations and/or justifications</td>
<td>✷ Incomplete or inaccurate recommendations and/or justifications</td>
<td></td>
</tr>
<tr>
<td>✷ Clear and accurate explanation of whether one of the plans represents a direct variation</td>
<td>✷ Largely correct explanation of whether one of the plans represents a direct variation</td>
<td>✷ Partially correct explanation of whether one of the plans represents a direct variation</td>
<td>✷ Incomplete or inaccurate explanation of whether one of the plans represents a direct variation</td>
<td></td>
</tr>
</tbody>
</table>
Learning Targets:

- Write the equation of a line in slope-intercept form.
- Use slope-intercept form to solve problems.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Marking the Text, Discussion Groups

When a diver descends in a lake or ocean, pressure is produced by the weight of the water on the diver. As a diver swims deeper into the water, the pressure on the diver’s body increases at a rate of about 1 atmosphere of pressure per 10 meters of depth. The table and graph below represent the total pressure, $y$, on a diver given the depth, $x$, under water in meters.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
</tr>
<tr>
<td>4</td>
<td>1.4</td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
</tr>
<tr>
<td>6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

1. Write an equation describing the relationship between the pressure exerted on a diver and the diver’s depth under water.

2. What is the slope of the line? What are the units of the slope?

3. What is the $y$-intercept? Explain its meaning in this context.

4. Identify the slope and $y$-intercept of the line described by the equation $y = -2x + 9$. 

**Slope-Intercept Form of a Linear Equation**

$$y = mx + b$$

where $m$ is the slope of the line and $(0, b)$ is the $y$-intercept.

**Pressure**

Pressure is force per unit area. Atmospheric pressure is defined using the unit atmosphere. 1 atm is 14.6956 pounds per square inch.

**Math Terms**

A **linear equation** is an equation that can be written in standard form $Ax + By = C$ where $A$, $B$, and $C$ are constants and $A$ and $B$ cannot both be zero.

**Math Tip**

Linear equations can be written in several forms.
5. Create a table of values for the equation \( y = -2x + 9 \). Then plot the points and graph the line.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

6. Explain how to find the value of the slope from the table. What is the value of the slope of the line?

7. Explain how to find the \( y \)-intercept from the table. What is the \( y \)-intercept?

8. Explain how to find the value of the slope from the graph. What is the value of the slope?

9. Explain how to find the \( y \)-intercept from the graph. What is the \( y \)-intercept?
Lesson 12-1
Slope-Intercept Form

Check Your Understanding

10. What are the slope and y-intercept of the line described by the equation $y = -\frac{4}{5}x - 10$?

11. Write the equation in slope-intercept form of the line that is represented by the data in the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>9</td>
</tr>
<tr>
<td>-1</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>

12. Write the equation, in slope-intercept form, of the line with a slope of 4 and a y-intercept of (0, 5).

13. Write an equation of the line graphed in the My Notes section of this page.

Monica gets on an elevator in a skyscraper. The elevator starts to move at a rate of $-20$ ft/s. After 6 seconds on the elevator, Monica is 350 feet from the ground floor of the building.

14. The rate of the elevator is negative. What does this mean in the situation? What value in the slope-intercept form of an equation does this rate represent?

15. a. How many feet was Monica above the ground when she got on the elevator? Show how you determined your answer.

b. What value in the slope-intercept form does your answer to Part (a) represent?

16. **Model with mathematics.** Write an equation in slope-intercept form for the motion of the elevator since it started to move. What do $x$ and $y$ represent?

a. What does the $y$-intercept represent?

b. Use the equation you wrote to determine, at this rate, how long it will take after Monica enters the elevator for her to exit the elevator on the ground floor. Explain how you found your answer.
Lesson 12-1
Slope-Intercept Form

Check Your Understanding

17. Write the equation $3x - 2y = 16$ in slope-intercept form. Explain your steps.

18. A flowering plant stands 6.5 inches tall when it is placed under a growing light. Its growth is 0.25 inches per day. Today the plant is 11.25 inches tall.
   a. Write an equation in slope-intercept form for the height of the plant since it was placed under the growing light.
   b. In your equation, what do $x$ and $y$ represent?
   c. Use the equation to determine how many days ago the plant was placed under the light.

LESSON 12-1 PRACTICE

19. What are the slope, $m$, and $y$-intercept, $(0, b)$, of the line described by the equation $3x + 6y = 12$?

20. Write an equation in slope-intercept form for the line that has a slope of $\frac{2}{3}$ and $y$-intercept of $(0, -5)$.

21. Write an equation in slope-intercept form for the line that passes through the points $(6, -3)$ and $(0, 2)$.

22. Matt sells used books on the Internet. He has a weekly fee he has to pay for his website. He has graphed his possible weekly earnings, as shown.
   a. What is the weekly fee that Matt pays for his website? How do you know?
   b. How much does Matt make for each book sold? How do you know?
   c. Write the equation in slope-intercept form for the line in Matt’s graph.
   d. How many books does Matt have to sell to make $30 for the week? Explain.

23. Make use of structure. Without graphing, describe the graph of each equation below. Tell whether the line is ascending or descending from left to right and where the line crosses the $y$-axis.
   a. $y = 3x$
   b. $y = 5x + 2$
   c. $y = -2x - 5$
   d. $y = -6x + 4$
Learning Targets:
• Write the equation of a line in point-slope form.
• Use point-slope form to solve problems.

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Note Taking, Think-Pair-Share, Critique Reasoning, Sharing and Responding

Another form of the equation of a line is the point-slope form. The point-slope form of the equation is found by solving the slope formula for \( y - y_1 \), by multiplying both sides by \( x - x_1 \). You may use this form when you know a point on the line and the slope.

Point-Slope Form of a Linear Equation

\[
y - y_1 = m(x - x_1)
\]

where \( m \) is the slope of the line and \((x_1, y_1)\) is a point on the line.

Example A

Write an equation of the line with a slope of \( \frac{1}{2} \) that passes through the point \((2, 5)\). Graph the line.

**Step 1:** Substitute the given values into point-slope form.

\[
y - y_1 = m(x - x_1) \\
y - 5 = \frac{1}{2}(x - 2)
\]

**Step 2:** Graph \( y - 5 = \frac{1}{2}(x - 2) \). Plot the point \((2, 5)\) and use the slope to find another point.

In calculus, the point-slope form of a line is used to write the equation of the line tangent to a curve at a given point.

MATH TIP

If you needed to express the solution to Example A in slope-intercept form, you could apply the Distributive Property and combine like terms.
Try These A
Find an equation of the line given a point and the slope.

a. \((-2, 7), \ m = \frac{2}{3}\)  
b. \((6, -1), \ m = -\frac{5}{4}\)

Determine the slope and a point on the line for each equation.

c. \(y + 4 = \frac{3}{8}(x - 3)\)  
d. \(y - 6 = -\frac{5}{2}(x + 3)\)

The town of San Simon charges its residents for trash pickup and water usage on the same bill. Each month the city charges a flat fee for trash pickup and a fee of $0.25 per gallon for water used. In January, one resident used 44 gallons of water, and received a bill for $16.

1. If \(x\) is the number of gallons of water used during a month, and \(y\) represents the bill amount in dollars, write a point \((x_1, y_1)\).

2. What does $0.25 per gallon represent?

3. **Reason abstractly.** Use point-slope form to write an equation that represents the bill cost \(y\) in terms of the number of gallons of water \(x\) used in a month.

4. Write the equation in Item 3 in slope-intercept form. What does the \(y\)-intercept represent?

Check Your Understanding

5. Determine the equation of the line given the point \((86, 125)\) and the slope \(m = -18\).

6. Violet has an Internet business selling paint sets. After an initial website fee each week, she makes a profit of $0.75 on each set she sells. If she sells 8 sets, she makes $2.25. Write an equation representing her weekly possible earnings.
Lesson 12-2
Point-Slope Form

7. Critique the reasoning of others. Jamilla and Ryan were asked to write the equation of the line through the points (6, 4) and (3, 5). Both Jamilla and Ryan determined that the slope was $-\frac{1}{3}$. Jamilla wrote the equation of the line as $y - 4 = -\frac{1}{3}(x - 6)$. Ryan wrote the equation of the line as $y - 5 = -\frac{1}{3}(x - 3)$.

a. Rewrite each student’s equation in slope-intercept form and compare the results.

b. Whose equation was correct? Justify your response.

8. Find the equation in point-slope form of the line shown in the graph.

9. Write the equation of the line in slope-intercept form.

Check Your Understanding

10. Explain the process you would use to write an equation of a line in point-slope form when given two points on the line.

11. Describe the similarities and differences between point-slope form and slope-intercept form.
LESSON 12-2 PRACTICE

12. Write an equation of the line with a slope of 0.25 that passes through the point \((-1, -8)\).

13. Find the slope and a point on the line for the lines with the following equations.
   a. \(y - 9 = -\frac{3}{4}(x - 4)\)
   b. \(y = 3 - \frac{2}{3}(x + 4)\)

14. Write the equation of the line through the points \((-3, 3)\) and \((7, 5)\) in slope-intercept form. What is the \(y\)-intercept?

15. Jay pays a flat fee each month for basic cable service. He also pays $3.50 for each movie he orders during the month. Last month, he ordered 5 movies and his total bill came to $54.
   a. Write an equation in point-slope form that represents the total bill, \(y\), in terms of the number of movies, \(x\).
   b. Write the equation in slope-intercept form.
   c. What is the monthly fee for basic cable service? How do you know?
   d. Next month, Jay plans to order 7 movies. What will be his total bill for the month?
   e. This month, Jay’s total bill is $78.50. How many movies did he order this month?

16. Attend to precision. The equation \(y - 160 = 40(x - 1)\) represents the height in feet, \(y\), of a hot-air balloon \(x\) minutes after the pilot started her stopwatch.
   a. Is the hot-air balloon rising or descending? Justify your answer.
   b. At what rate is the hot-air balloon rising or descending? Be sure to use appropriate units.
   c. What was the height of the balloon when the pilot started her stopwatch?
Learning Targets:
- Write the equation of a line in standard form.
- Use the standard form of a linear equation to solve problems.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Note Taking, Discussion Groups, Think-Pair-Share, Identify a Subtask

A linear equation can be written in the form $Ax + By = C$ where $A$, $B$, and $C$ are constants and $A$ and $B$ are not both zero.

**Standard Form of a Linear Equation**

$$Ax + By = C$$

where $A \geq 0$, $A$ and $B$ are not both zero, and $A$, $B$, and $C$ are integers whose greatest common factor is 1.

1. **Reason abstractly.** You can use the coefficients of this form of an equation to find the $x$-intercept, $y$-intercept, and slope.
   a. Determine the $x$-intercept.

   b. Determine the $y$-intercept.

   c. Write $Ax + By = C$ in slope-intercept form to find the slope.

   The definition of standard form states that both $A$ and $B$ are not 0. However, one of $A$ or $B$ may be equal to 0.

2. **Write the standard form if $A = 0$.**
   a. Suppose $A = 0$, $B = -1$, and $C = 3$. Write the equation of the line in standard form.

   b. Graph the line on the grid in the My Notes section. Describe the graph. What is the slope?

3. **Write the standard form if $B = 0$.**
   a. Suppose $A = 1$, $B = 0$, and $C = -6$. Write the equation of the line in standard form.

   b. Graph the equation on the grid in the My Notes section. Describe the graph. What is the slope?
4. Write $3x + 2y = 8$ in slope-intercept form.

5. Write the equation $y - 7 = 2(x + 1)$ in standard form.

**Check Your Understanding**

6. Write the equation $2x + 3y = 18$ in slope-intercept form.

7. Write the equation $y = \frac{6}{5}x - 4$ in standard form.

8. Describe the graph of any line whose equation, when written in standard form, has $A = 0$.

9. Susheila is making a large batch of granola to sell at a school fundraiser. She needs to buy walnuts and almonds to make the granola. Walnuts cost $3 per pound and almonds cost $2 per pound. She has $30 to spend on these ingredients.

   a. Write an equation that represents the different amounts of walnuts, $x$, and almonds, $y$, that Susheila can buy.

   b. Graph the $x$- and $y$-intercepts on the coordinate plane below. Use these to help you graph the line.

   c. If Susheila buys 4 pounds of walnuts, how many pounds of almonds can she buy?
10. Refer to the graph you made in Item 9b. What is the $x$-intercept? What does it represent?

11. Write an equation in standard form for the line shown.

12. Make use of structure. The equation $2x - 5y = 20$, the table below, and the graph below represent three different linear functions.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-1</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
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<tr>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>19</td>
</tr>
</tbody>
</table>

Which function represents the line with the greatest slope? Explain your reasoning.
Lesson 12-3
Standard Form

Check Your Understanding

13. Write an equation in standard form for the line that is represented by the data in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

14. Write an equation in standard form for the line with a slope of 7 that passes through the point (1, 2).

LESSON 12-3 PRACTICE

15. Determine the x-intercept, y-intercept, and slope of the line described by \(-3x + 7y = -21\).

16. Write each equation in standard form.
   a. \(8x = 26 + 14y\)
   b. \(y = \frac{6}{7}x + 12\)

17. Write an equation in standard form for each line below.
   a. 
   b. 

18. Pedro walks at a rate of 4 miles per hour and runs at a rate of 8 miles per hour. Each week, his exercise program requires him to cover a total distance of 20 miles with some combination of walking and/or running.
   a. Write an equation that represents the different amounts of time Pedro can walk, x, and run, y, each week.
   b. Graph the equation.
   c. What is the y-intercept? What does this tell you?

19. Make sense of problems. Keisha bought a discount pass at a movie theater. It entitles her to a special discounted admission price for every movie she sees. Keisha wrote an equation that gives the total cost y of seeing x movies. In standard form, the equation is \(7x - 2y = -31\).
   a. What was the cost of the pass?
   b. What is the discounted admission price for each movie?
Learning Targets:

- Describe the relationship among the slopes of parallel lines and perpendicular lines.
- Write an equation of a line that contains a given point and is parallel or perpendicular to a given line.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Discussion Groups

Parallel lines and perpendicular lines are pairs of lines that have special relationships.
Parallel lines in a plane are equidistant from each other at all points.

1. Consider lines $l_1$, $l_2$, $l_3$, and $l_4$ on the graph above. Determine the slope of each line.

2. Reason quantitatively. In the graph above, $l_1$ is parallel to $l_2$ and $l_3$ is parallel to $l_4$. Write a conjecture about the slopes of parallel lines.

3. Determine the slope of a line that is parallel to the line whose equation is $y = -3x + 4$.

4. Write the equation of a line that is parallel to the line $y = \frac{3}{4}x - 1$ and has a $y$-intercept of $(0, 5)$. 
5. Horizontal lines are described by equations of the form \( y = \text{number} \). For example, the equation of the \( x \)-axis is \( y = 0 \), because all points on the \( x \)-axis have \( y \)-coordinate 0. Explain why any two horizontal lines are parallel.

6. Vertical lines are described by equations of the form \( x = \text{number} \). For example, the equation of the \( y \)-axis is \( x = 0 \), because all points on the \( y \)-axis have \( x \)-coordinate 0. Do you think that any two vertical lines are parallel? Explain why or why not.

7. Use the information in Items 5 and 6 to write the equation of a line that is
   a. parallel to the \( x \)-axis.
   b. parallel to the \( y \)-axis.

8. A line is parallel to \( y = 3x + 2 \) and passes through the point (1, 4).
   a. What is the slope of the line? Explain how you know.
   b. Write an equation of the line.

9. Graph and label each line described below on the grid in the My Notes section. Which lines appear to be perpendicular?
   - \( l_5 \) has slope \(-\frac{4}{3}\) and contains the point (0, 2).
   - \( l_6 \) has slope \(-\frac{3}{4}\) and contains the point (0, 0).
   - \( l_7 \) has slope \(\frac{3}{4}\) and contains the point (−2, −1).

10. Write a conjecture about the slopes of perpendicular lines.
Lesson 12-4
Slopes of Parallel and Perpendicular Lines

11. Use your prediction from Item 10 to write the equations of two lines that are perpendicular. On the grid in the My Notes section on the previous page, graph both lines and confirm that they are perpendicular.

12. In the coordinate plane, what is true about a line that is perpendicular to a horizontal line?

13. Line $l_1$ contains the points (0, −1) and (3, 1). It is perpendicular to line $l_2$ that contains the point (−1, 2).
   a. What is the slope of each line? Explain how you know.
   b. Write the equation of each line.

14. Determine whether the lines with the given slopes are parallel, perpendicular, or neither.
   a. $m_1 = -4$, $m_2 = \frac{1}{4}$
   b. $m_1 = -3$, $m_2 = 3$
   c. $m_1 = \frac{10}{12}$, $m_2 = -1\frac{1}{2}$
   d. $m_1 = \frac{1}{2}$, $m_2 = \frac{1}{2}$

15. The equation of line $l_1$ is $y = \frac{1}{3}x - 2$.
   a. Write the equation of a line parallel to $l_1$. Explain.
   b. Write the equation of a line perpendicular to $l_1$. Explain.

16. Write the equation of a line that is parallel to the line $3x + 4y = 4$ and contains the point (8, 1).

17. Write an equation of a line that is perpendicular to the line $y = 5x + 1$ and contains the point (−10, 2).
LESSON 12-4 PRACTICE

18. Determine whether the lines with the given slopes are parallel, perpendicular, or neither.
   a. \( m_1 = 5, m_2 = \frac{1}{5} \)
   b. \( m_1 = -6, m_2 = \frac{1}{6} \)
   c. \( m_1 = -\frac{2}{3}, m_2 = -\frac{2}{3} \)

19. The slopes of three lines are given below.
   \( m_1 = -\frac{1}{2} \)  \( m_2 = 3 \)  \( m_3 = 0 \)
   a. Determine the slope of a line that is parallel to a line with each given slope.
   b. Determine the slope of a line that is perpendicular to a line with each given slope.

20. Determine the slope of any line that is parallel to the line described by \( y = -\frac{1}{2}x + 5 \).

21. Write the equation of a line that is parallel to the line described by \( x - 4y = 8 \). Explain how you know the lines are parallel.

22. Determine the slope of any line that is perpendicular to the line described by \( y = \frac{3}{4}x - 9 \).

23. Write an equation of the line that is perpendicular to the line \( 2x + 5y = -15 \) and contains the point \((-8, 3)\).

24. Determine the equation of a line perpendicular to the \( x \)-axis that passes through the point \((4, -1)\).

25. Construct viable arguments. A line \( a \) passes through points with coordinates \((-3, 5)\) and \((0, 0)\) and a line \( b \) passes through points with coordinates \((3, 5)\) and \((0, 0)\). Are lines \( a \) and \( b \) parallel, perpendicular, or neither? Explain your answer.
**ACTIVITY 12 PRACTICE**

Write your answers on notebook paper. Show your work.

**Lesson 12-1**

1. Write the equation of a line in slope-intercept form that has a slope of \(-8\) and a \(y\)-intercept of \((0, 3)\).

2. Write the equation of a line in slope-intercept form that passes through the point \((0, -7)\) and has a slope of \(\frac{3}{4}\).

3. Find the slope and the \(y\)-intercept of the line whose equation is \(-5x + 3y - 8 = 0\).

4. Which of the following is the slope-intercept form of the equation of the line in the graph?

   A. \(y = -\frac{5}{3}x + 3\)
   
   B. \(y = -\frac{3}{5}x + 5\)
   
   C. \(y = -\frac{3}{5}x + 3\)
   
   D. \(y = -\frac{5}{3}x + 5\)

5. What is the initial fee Mike pays each week?

6. How many packs does Mike have to sell to break even?

7. What is the price of one pack of cards?

8. What is the equation in slope-intercept form for the line shown in graph?

9. How many packs of cards must Mike sell to make $40? Explain.

**Lesson 12-2**

10. What is the equation in point-slope form of the line that passes through \((-9, 12)\) with a slope of \(\frac{5}{6}\)?

11. What is the equation in slope-intercept form of the line that has a slope of 0.25 and passes through the point \((6, -8)\)?

12. What is the equation in point-slope form of the line that passes through the points \((2, -3)\) and \((-5, 8)\)?
13. Write an equation in slope-intercept form of the line that passes through the points (4, 2) and (1, −7).

14. What is the equation in slope-intercept form of the line that passes through the points (2, 7) and (6, 7)? Describe the line.

15. What is the point-slope form of the line in the graph?

\begin{align*}
\text{Graph of a line}
\end{align*}

Lesson 12-3

16. Write the equation of the line in the graph from Item 15 in standard form.

17. David is ordering tea from an online store. Black tea costs $0.80 per ounce and green tea costs $1.20 per ounce. He plans to spend a total of $12 on the two types of tea.

a. Write an equation that represents the different amounts of black tea, \(x\), and green tea, \(y\), that David can buy.

b. Graph the equation.

c. What is the \(x\)-intercept? What does it represent?

d. Suppose David decides to buy 10 ounces of black tea. How many ounces of green tea will he buy?

18. Is the equation \(6x - 15y = -12\) in standard form? Why or why not?

19. Which is a true statement about the line \(x - 4y = 8\)?

A. The \(x\)-intercept of the line is (2, 0).

B. The \(y\)-intercept of the line is (0, 2).

C. The slope of the line is \(\frac{1}{4}\).

D. The line passes through the origin.

20. Write the equation of a line in standard form that has an \(x\)-intercept of (3, 0) and a \(y\)-intercept of (0, 5).

Lesson 12-4

21. What is the slope of a line parallel to a line whose equation is \(3x + 5y = 12\)?

22. What is the slope of a line perpendicular to a line whose equation is \(-4x - 2y + 18 = 0\)?

23. Which is the slope of a line that is perpendicular to the line whose equation is \(5x - 3y = -10\)?

A. \(\frac{3}{5}\)

B. \(-\frac{3}{5}\)

C. \(\frac{5}{3}\)

D. \(-\frac{5}{3}\)

24. What is the equation of the line that is perpendicular to \(2x + 4y = 1\) and that passes through the point (6, 8)?

25. What is the slope of any line that is perpendicular to the line that contains the points (8, 8) and (12, 12)?

MATHEMATICAL PRACTICES

Construct Viable Arguments and Critique the Reasoning of Others

26. Aidan stated that for any value of \(b\), the line \(y = 2x + b\) is parallel to the line that passes through (2, 5) and (−1, −1). Do you agree with Aidan? Explain why or why not.
Learning Targets:
- Use collected data to make a scatter plot.
- Determine the equation of a trend line.

**SUGGESTED LEARNING STRATEGIES:** Predict and Confirm, Sharing and Responding, Create Representations, Look for a Pattern, Interactive Word Wall

How fast can you and your classmates pass a textbook from one person to the next until the book has been relayed through each person in class?

1. Suppose your entire class lined up in a row. Estimate the length of time you think it would take to pass a book from the first student in the row to the last. Assume that the book starts on a table and the last person must place the book on another table at the end of the row.

   Estimated time to pass the book: _____________

2. As a class, experiment with the actual time it takes to pass the book using small groups of students in your class. Use the table below to record the times.

<table>
<thead>
<tr>
<th>Number of students passing the book</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>11</th>
<th>13</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to pass the book (nearest tenth of a second)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. **Reason quantitatively.** Based on the data you recorded in the table above, would you revise your estimated time from Item 1? Explain the reasoning behind your answer.
4. Graph the data in your table from Item 2 as a scatter plot on the coordinate grid.

5. Are the data that you collected linear data?
   a. Explain your answer using the scatter plot.
   b. Explain your answer using the table of data.

6. Describe how the time to pass the book changes as the number of students increases.

7. Work as a group to predict the number of seconds it will take to pass the book through the whole class.
   a. Place a trend line on the scatter plot in Item 4 in a position that your group feels best models the data. Then, mark two points on the line.
   b. In the spaces provided below, enter the coordinates of the two points identified in Part (a).

   Point 1: (______, ______)  Point 2: (______, ______)

   c. Why does your group think that this line gives the best position for modeling the scatter plot data?
Lesson 13-1
Scatter Plots and Trend Lines

8. Use the coordinate pairs you recorded in Item 7b to write the equation for your trend line (or linear model) of the scatter plot.

9. Explain what the variables in the equation of your linear model represent.

10. **Reason abstractly.** Interpret the meaning of the slope in your linear model.

11. Use your model to predict how long it would take to pass the book through all the students in your class.

   Predicted time to pass the book: ______________

12. Using all of the students in your class, find the actual time it takes to pass the book.
   
   Actual time to pass the book: ______________

13. How do your estimate from Item 1 and your predicted time from Item 11 compare to the actual time that it took to pass the book through the entire class?

14. **Attend to precision.** Suppose that another class took 1 minute and 47 seconds to pass the book through all of the students in the class. Use your linear model to estimate the number of students in the class.
Check Your Understanding

The table shows the number of days absent and the grades for several students in Ms. Reynoso’s Algebra 1 class.

<table>
<thead>
<tr>
<th>Days Absent</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade (percent)</td>
<td>98</td>
<td>88</td>
<td>69</td>
<td>89</td>
<td>90</td>
<td>86</td>
<td>77</td>
</tr>
</tbody>
</table>

15. Create a scatter plot of the data using Days Absent as the independent variable.

16. Are the data linear? Explain using the scatter plot and the table of data.

17. Based on the data, how do grades change as the number of days absent increases?

18. Draw a trend line on your scatter plot. Identify two points on the trend line and write an equation for the line containing those two points.

19. What is the meaning of the \( x \) and \( y \) variables in the equation you wrote?

20. Interpret the meaning of the slope and the \( y \)-intercept of your trend line.

21. Use your equation to predict the grade of a student who is absent for 5 days.

LESSON 13-1 PRACTICE

Model with mathematics. The scatter plot shows the day of the month and total rainfall for January.

22. Copy the scatter plot and draw a trend line on the scatter plot. Identify two points on the trend line and write a linear equation to model the data containing those two points.

23. Explain the meaning of \( x \) and \( y \) in your equation.

24. Interpret the meaning of the slope and the \( y \)-intercept of your trend line.
Lesson 13-2
Linear Regression

Learning Targets:
- Use a linear model to make predictions.
- Use technology to perform a linear regression.

**SUGGESTED LEARNING STRATEGIES:** Marking the Text, Interactive Word Wall, Look for a Pattern, Think-Pair-Share, Quickwrite

There is a *correlation* between two variables if they share some kind of relationship.

1. Is there a correlation between the variables of your linear model in Item 4 in Lesson 13-1? Explain.

Examples of data with two variables that illustrate a *positive correlation*, a *negative correlation*, and *no correlation* are shown below. The more closely the data resemble a line, the stronger the linear correlation.

2. Look back at your linear model in Item 4 in Lesson 13-1. Does your linear model represent a positive correlation, a negative correlation, or no correlation? Explain.

There is *causation* between two variables if a change in one variable causes the other variable to change. For example, doing more exercise causes a greater number of calories to be burned.

3. Does there seem to be causation between the variables of your linear model in Item 4 in Lesson 13-1? Explain.

**MATH TERMS**
A scatter plot will show a positive correlation if \( y \) tends to increase as \( x \) increases. Other data may have a negative correlation, where \( y \) tends to decrease as \( x \) increases, or no correlation. A correlation is sometimes called an association.

**ACADEMIC VOCABULARY**
The idea of causation is important in physics. For example, a cause can be represented by a force acting on an object.
Correlation does not imply causation. Just because there is a correlation between two variables does not mean that there is causation between them; there may be other factors affecting the situation.

**Check Your Understanding**

4. Consider the following two variables: your shoe size each year since you were born and the average price of a movie ticket each year since you were born.
   a. Is there a correlation between the variables? Explain.
   b. Is there causation between the variables? Explain.

5. Give an example of two variables for which there is both correlation and causation.

A scatter plot and a **line of best fit**, the most accurate trend line, can be created using a graphing calculator, a spreadsheet program, or other Computer Algebra Systems (CAS).

**Linear regression** is a method used to find the line of best fit. A line found using linear regression is more accurate than a trend line that has been visually estimated. You can perform linear regression using a graphing calculator.

6. **Use appropriate tools strategically.** Enter the book-passing data you collected in Item 2 in Lesson 13-1 into your graphing calculator. Enter the numbers of students as *x*-values and the corresponding times to pass the book as *y*-values.

   a. To find the equation of the line of best fit, use the linear regression feature of your calculator.

   The calculator should return values for *a* and *b*. Write these values below.

   \[ a = \]

   \[ b = \]

   b. The value of *a* is the slope of the line of best fit, and (0, *b*) is the *y*-intercept. Round *a* and *b* to the nearest hundredth and write the equation of the line of best fit in the form \[ y = ax + b. \] Describe how this equation is different from or similar to your equation in Item 8 in Lesson 13-1.
Lesson 13-2
Linear Regression

Check Your Understanding

7. Enter the following data into your graphing calculator. Make sure that any previous data have been cleared.
   \((6, 1), (9, 0), (12, -3), (3, 3), (0, 5), (-3, 7), (-5, 9), (-7, 13)\)
   
   a. Find the equation of the line of best fit. Round values to the nearest hundredth.
   b. Use the equation of the line of best fit to predict the value of \(y\) when \(x = 20\).

8. Compare using a graphing calculator to using paper and pencil when plotting data and finding a trend line.

LESSON 13-2 PRACTICE

The owner of a café kept records on the daily high temperature and the number of hot apple ciders sold on that day. Some of the owner’s data are shown below.

<table>
<thead>
<tr>
<th>Daily High Temperature (°F)</th>
<th>32</th>
<th>75</th>
<th>80</th>
<th>48</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Hot Apple Ciders Sold</td>
<td>51</td>
<td>22</td>
<td>12</td>
<td>40</td>
<td>70</td>
</tr>
</tbody>
</table>

9. Create a scatter plot of the data.

10. Is there a correlation between the variables? If so, what type?

11. Determine the equation of the line of best fit. Round values to the nearest hundredth.

12. What is the slope? What does the slope represent?

13. Identify the \(y\)-intercept. What does the \(y\)-intercept represent?

14. **Model with mathematics.** Use your model to predict the number of hot apple ciders the café would sell on a day when the high temperature is 92°F. Explain.
Learning Targets:
- Use technology to perform quadratic and exponential regressions, and then make predictions.
- Compare and contrast linear, quadratic, and exponential regressions.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Quickwrite, Think-Pair-Share, Discussion Groups

Online shopping has experienced tremendous growth since the year 2000. One way to measure the growth is to track the average number of daily hits at the websites of online stores. The tables show the average number of daily hits for three different online stores in various years since the year 2000.

<table>
<thead>
<tr>
<th>Nile River Retail</th>
<th>Years Since 2000</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Daily Hits (thousands)</td>
<td>52.1</td>
<td>56.2</td>
<td>60.0</td>
<td>64.1</td>
<td>68.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>eBuy</th>
<th>Years Since 2000</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Daily Hits (thousands)</td>
<td>1.0</td>
<td>4.9</td>
<td>17.2</td>
<td>37.2</td>
<td>64.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spendco</th>
<th>Years Since 2000</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Daily Hits (thousands)</td>
<td>2.0</td>
<td>6.6</td>
<td>20.9</td>
<td>68.1</td>
<td>220.4</td>
</tr>
</tbody>
</table>

1. Make sense of problems. Compare and contrast the growth of the three online stores based on the data in the tables.
2. Plot the data for the three online stores on the graphs below.

**Nile River Retail**

**eBuy**

**Spendco**
3. Which online store’s growth could best be modeled by a linear function? Explain.

4. For the online store you identified in Item 3, determine the equation of the line of best fit. Round values to the nearest hundredth.

5. Predict the number of daily hits for this online store in 2015.

When a line does not appear to be a good fit for a set of data, you may want to model the data using a nonlinear model.

**Quadratic regression** is a method used to find a **quadratic function** that models a set of data. You can perform quadratic regression using a graphing calculator.

6. Enter the data for eBuy into your graphing calculator. Enter the years since 2000 as the x-values and the corresponding daily hits in thousands as the y-values.
   a. To find the quadratic equation that models the data, use the quadratic regression feature of your calculator.

   The calculator should return values for \(a\), \(b\), and \(c\). Write these values below, rounding to the nearest hundredth.
   
   \[ a = \]
   \[ b = \]
   \[ c = \]

   b. Write the quadratic equation in the form \(y = ax^2 + bx + c\).

   c. Use the quadratic equation to predict the number of daily hits for eBuy in 2015.
Lesson 13-3
Quadratic and Exponential Regressions

When a set of data shows very rapid growth or decay, an exponential model may be the best choice for modeling the data.

**Exponential regression** is a method used to find an *exponential function* that models a set of data. You can perform exponential regression using a graphing calculator.

7. Enter the data for Spendco into your graphing calculator. Enter the years since 2000 as the $x$-values and the corresponding daily hits in thousands as the $y$-values.
   a. To find the exponential equation that models the data, use the exponential regression feature of your calculator.
      The calculator should return values for $a$ and $b$. Write these values below, rounding to the nearest hundredth.
      
      $a = $

      $b = $  

   b. Write the exponential equation in the form $y = ab^x$.

   c. Use the exponential equation to predict the number of daily hits for Spendco in 2015.

8. **Construct viable arguments.** Based on your predictions for the number of daily hits for each online store in 2015, which type of function has the fastest growth: linear, quadratic, or exponential? Explain.
Lesson 13-3 Practice

The population of Williston, North Dakota, has grown rapidly over the past decade due to an oil boom. The table gives the population of the town in 2007, 2009, and 2011.

<table>
<thead>
<tr>
<th>Years Since 2000</th>
<th>7</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (thousands)</td>
<td>12.4</td>
<td>13.0</td>
<td>16.0</td>
</tr>
</tbody>
</table>

11. Use your calculator to find the equation of the line of best fit for the data.

12. **Reason quantitatively.** What is the slope of the line? What does it tell you about the population growth of the town?

13. Use your calculator to find a quadratic equation that models the growth of the town.

14. Use your quadratic equation to predict the population of Williston in 2020.

15. Use your calculator to find an exponential equation that models the growth of the town.

16. Use your exponential equation to predict the population of Williston in 2020.

17. According to the exponential model, in what year will the town have a population greater than 40,000 for the first time? *(Hint: Use the table feature of your calculator.)* What assumptions do you make when you use the exponential model to answer this question?

9. How are quadratic regression and exponential regression similar to and different from linear regression?

10. Do you think the exponential model would be appropriate for predicting the number of daily hits for Spendco in any future year? Explain your reasoning.
ACTIVITY 13 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 13-1
The scatter plot shows the relationship between the day of the month and a frozen yogurt stand’s daily profit during the month of the July.

1. Are the data linear? Explain.
2. Draw a trend line on the scatter plot and name two points that your trend line passes through.
3. Write the equation of the trend line you drew in Item 2.
4. What do the variables in your equation represent?
5. What is the slope of the trend line? What does this tell you?
6. Use your trend line to predict the yogurt stand’s daily profit on July 20.
7. The owner of a competing frozen yogurt stand finds that her daily profit each day in July is exactly $100 more than that of the stand in the scatter plot. Write the equation of a trend line for the competing stand.
8. The manager of a local history museum experiments with different prices for admission to the museum. For each price, the manager notes the number of visitors who enter the museum on that day. The table shows the data.

<table>
<thead>
<tr>
<th>Price</th>
<th>$2.75</th>
<th>$3.50</th>
<th>$4.25</th>
<th>$5.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Daily Visitors</td>
<td>112</td>
<td>88</td>
<td>66</td>
<td>63</td>
</tr>
</tbody>
</table>

Which is a true statement about the data?
A. A trend line on the scatter plot has a positive slope.
B. The y-intercept of the trend line is above the x-axis.
C. The trend line predicts at least 70 visitors when the admission price is $6.25.
D. The trend line fits the data perfectly because the data is linear.

Lesson 13-2
Use your calculator to perform a linear regression for the following data. Use your linear regression for Items 9–12.

$(-6, -3), (-8, -4), (-2, 1), (1, 4), (3, 6), (5, 8), (7, 13)$

9. What is the equation of the line of best fit?
10. What is the value of the slope? What does this tell you about the relationship between $x$ and $y$?
11. According to your model, what is the value of $y$ when $x = -19$?
12. According to your model, for what value of $x$ is $y = 100$?
13. Look at the scatter plot on the previous page showing the daily profits of a frozen yogurt stand. What type of correlation, if any, does the scatter plot show?

14. Which of the following pairs of variables are likely to show a negative correlation?
   A. the length of a shoe; the size of the shoe
   B. the number of miles on a car’s odometer; the age of the car
   C. the weight of a watermelon; the price of the watermelon
   D. the number of minutes you have waited for a bus; the number of minutes remaining until the bus arrives

15. At several times during the school year, Emilio collected data on the height of a plant in the classroom and the total number of quizzes he had taken so far in his science class. The data are shown below.

<table>
<thead>
<tr>
<th>Height of Plant (cm)</th>
<th>16</th>
<th>19</th>
<th>22</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Quizzes</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

a. Is there a correlation between the variables? Explain.
b. Is there causation between the variables? Explain.

Lesson 13-3
The table shows the number of employees at a software company in various years.

<table>
<thead>
<tr>
<th>Years Since 2000</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Employees</td>
<td>32</td>
<td>40</td>
<td>75</td>
<td>124</td>
</tr>
</tbody>
</table>

16. Make a scatter plot of the data.

17. Do you think a linear equation would be a good model for the data? Justify your answer.

18. Use your calculator to find a quadratic equation that models the growth of the company.

19. Use the quadratic model to predict the number of employees in the year 2015.

20. Use your calculator to find an exponential equation that models the growth of the company.

21. Use the exponential model to predict the number of employees in the year 2015.

22. How do the predictions given by the two models in Items 19 and 21 compare?

23. Use your calculator to compare the quadratic and exponential models. Enter the equation from Item 19 as \( Y_1 \) and the equation from Item 21 as \( Y_2 \). View the graphs in a window that allows you to compare their growth. What do you notice?

The table shows the total number of bacteria in a sample over five hours.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Number of Bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>144</td>
</tr>
<tr>
<td>3</td>
<td>1728</td>
</tr>
<tr>
<td>4</td>
<td>20,736</td>
</tr>
<tr>
<td>5</td>
<td>248,832</td>
</tr>
</tbody>
</table>

24. Use your calculator to find an exponential equation that models the bacteria data.

25. If this trend continues, how many bacteria will be growing in the sample after 9 hours?

MATHEMATICAL PRACTICES
Look for and Make Use of Structure

26. Is it possible to tell from the equation of a line of best fit whether there is a positive or negative correlation between two variables? If so, explain how. If not, explain why not.
Jim was serving as a finish-line judge for the Striders 10K Run. He was interested in finding out how three of his friends were doing out on the course. He was able to get the following data from racing officials.

<table>
<thead>
<tr>
<th>Runner: J. Matuba</th>
<th>Time (min)</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>12</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (m)</td>
<td>1090</td>
<td>1380</td>
<td>2040</td>
<td>3640</td>
<td>6300</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Runner: E. Rodriguez</th>
<th>Time (min)</th>
<th>1</th>
<th>6</th>
<th>10</th>
<th>18</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (m)</td>
<td>500</td>
<td>2000</td>
<td>3280</td>
<td>5510</td>
<td>7700</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Runner: T. Donovan</th>
<th>Time (min)</th>
<th>2</th>
<th>4</th>
<th>9</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (m)</td>
<td>620</td>
<td>1250</td>
<td>2900</td>
<td>4690</td>
<td>6250</td>
<td></td>
</tr>
</tbody>
</table>

Answer Items 1–3 below, based on the information Jim received about his three running friends. Use $x$ as the number of minutes elapsed since the race began and $y$ as the number of meters completed.

1. Make a scatter plot showing the data for each runner.
2. Perform a linear regression to find the equation of the line of best fit for each runner. Round values in the equations to the nearest tenth.
3. Explain the order in which the runners will finish the race based on the models you formed using the data.

Answer the following questions for the linear models you formed. Explain your answers.

4. What is the standard form of the linear model for Matuba?
5. What is the domain of the linear model for Rodriguez?
6. What is the slope of the linear model for Donovan? What is its significance in the context of the problem situation?
### Scoring Guide

<table>
<thead>
<tr>
<th>Mathematics Knowledge and Thinking (Items 1, 2, 4–6)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Clear and accurate understanding of scatter plots, linear regression, standard form of a linear model, domain, and slope</td>
<td>• Largely correct understanding of scatter plots, linear regression, standard form of a linear model, domain, and slope</td>
<td>• Partial understanding of scatter plots, linear regression, standard form of a linear model, domain, and slope</td>
<td>• Inaccurate or incomplete understanding of scatter plots, linear regression, standard form of a linear model, domain, and slope</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Problem Solving (Item 3)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Appropriate and efficient strategy that results in a correct answer</td>
<td>• Strategy that may include unnecessary steps but results in a correct answer</td>
<td>• Strategy that results in a partially incorrect answer</td>
<td>• No clear strategy when solving problems</td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Mathematical Modeling / Representations (Items 1, 2, 4, 6)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Clear and accurate scatter plot</td>
<td>• Largely correct scatter plot</td>
<td>• Partially correct scatter plot</td>
<td>• Inaccurate or incomplete scatter plot</td>
<td></td>
</tr>
<tr>
<td>• Fluency in fitting a linear model to real-world data, including how to interpret and draw accurate conclusions from the model</td>
<td>• Adequate understanding of how to fit a linear model to real-world data, including how to interpret and draw accurate conclusions from the model</td>
<td>• Partial understanding of how to fit a linear model to real-world data, including how to interpret and draw accurate conclusions from the model</td>
<td>• Little or no understanding of how to fit a linear model to real-world data, including how to interpret and draw accurate conclusions from the model</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reasoning and Communication (Items 3–6)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Precise use of appropriate math terms and language to explain the order in which the runners will finish, including justification based on the model</td>
<td>• Adequate explanation and justification of the order in which the runners will finish</td>
<td>• Misleading or confusing explanation and justification of the order in which the runners will finish</td>
<td>• Incomplete or inaccurate explanation and justification of the order in which the runners will finish</td>
<td></td>
</tr>
<tr>
<td>• Clear and accurate descriptions of how to find the standard form, identify a reasonable domain, and identify and interpret the slope of a linear model</td>
<td>• Largely correct description of how to find the standard form, identify a reasonable domain, and identify and interpret the slope of a linear model</td>
<td>• Partially correct description of how to find the standard form, identify a reasonable domain, and identify and interpret the slope of a linear model</td>
<td>• Incorrect or incomplete description of how to find the standard form, identify a reasonable domain, and identify and interpret the slope of a linear model</td>
<td></td>
</tr>
</tbody>
</table>